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The Students' Visual Reasoning in Solving Integral Problems

Ummu Sholihah[1*], Beni Asyhar[2]
[1*,2] Program Studi Tadris Matematika, Universitas Islam Negeri Sayyid Ali Rahmatullah Tulungagung, Indonesia
**Penulis Korespondensi. Jl. Mayor Sujadi No. 46, 66221, Kab. Tulungagung, Indonesia*
E-mail: ummu_sholihah@uinsatu.ac.id [1)]*
asyhar_beni@yahoo.com [2)]

Keywords

Penalaran Visual, Menyelesaikan masalah, Integral.

ABSTRACT

Kalkulus integral merupakan salah satu mata kuliah matematika yang memerlukan penalaran, kemampuan komunikasi serta kemampuan berpikir tingkat tinggi dalam menyelesaikannya. Tujuan penelitian ini untuk mendeskripsikan penalaran visual mahasiswa dalam menyelesaikan masalah integral. Subjek penelitian 3 mahasiswa S1 Prodi Studi Tadris Matematika UIN Sayyid Ali Rahmatullah Tulungagung. Teknik pengumpulan data menggunakan tes dan wawancara. Analisis data dengan mereduksi data, menyajikan data dan menarik kesumpulan. Hasil penelitian menunjukkan adanya perbedaan penalaran mahasiswa laki-laki dan Perempuan dalam menyelesaikan masalah integral. Mahasiswa Laki-laki lebih mampu memenuhi proses penalaran yaitu representasi visual, visualisasi, dan transisi ke berpikir matematis.

Visual reasoning, Problem Solving, Integral

Integral calculus is a mathematics course that requires reasoning, communication skills and high-level thinking skills to complete it. The objective of this research is to describe students' visual reasoning in solving integral problems. The research subjects were 3 undergraduate students from the Tadris Mathematics Program at UIN Sayyid Ali Rahmatullah Tulungagung. Data collection techniques used tests, think-aloud, and interviews. Data analysis used reducing data, presenting data, and concluding. The research results showed that there are differences in the reasoning of male and female students in solving integral problems. Male students are better able to fulfill the reasoning process, such as visual representation, visualization, and transition to mathematical thinking.

INTRODUCTION

Mathematics as a scientific discipline appeared from human empirical experience which was processed rationally in cognitive structures, producing various mathematical concepts. The foundations of mathematics are obtained through a thought process known as logic. Therefore, the ability to think logically is important for students in learning and understanding mathematics (Cahyani, L. N., Shodiq, L. J., & Agustin 2022; Kholil 2018). In dayly life, we often have challenges, ranging from simple to complex problems. A problem is generally considered a task or situation that is difficult to overcome or control because of its complexity. Not all individuals experience and have problems, however, when haved with a problem, someone is expected to find a solution or solve it (Seel 2012 dan Moursund 2005).

Special considerations are required when resolving or solving problems. This kind of activity involves logical thinking activities. Reasoning can be defined as the process of concluding or the way of thinking used to make statements and reach a conclusion (Leighton 2004). Considering the importance of reasoning in problem-solving, several studies showed that students with a low level of reasoning generally have difficulties in solving problems because they are less able to connect various existing facts to reach the correct conclusion (Putri, D. K., Sulianto, J., & Azizah 2019). Reasoning ability in mathematics can be a key factor in achieving success in solving HOTS questions (Sholihah and Aini 2023). Developing problem-solving skills in mathematics learning is considered a challenging high-level mental activity (Suherman 2001) stated that problem solving is considered the most difficult thing for students to learn and for educators to teach. (Bailey 1996) describes problem-solving as a complex and high-level activity in a person's mental processes. Problem-solving is defined as skills in solving problems that include identifying data needs, establishing mathematical models, selecting and implementing the chosen method, explaining results, and verifying their accuracy (Susanto 2019).

Students' logical thinking abilities decrease when haved with high school-level mathematics problems. They have not been able to formulate assumptions from existing information, formulate strong arguments, provide justification or evidence to support solutions, and have not been able to draw the right conclusions (Aziz et al. 2020). In contrast to before, the results of research (Pandu, Y. K., & Suwarsono 2021), showed that students' ability to solve function limit problems shows good quality in mathematical reasoning. Visual reasoning, which is one type of reasoning, is the main focus, especially in dealing with visual representations. As a first step, all processes begin with attention to visual representation (Mehmet Ertürk GEÇİCİ 2021).

Visual reasoning abilities include skills in describing, changing, creating, conveying, documenting, and reflecting visual information (Hershkowitz, R., Tabach, M., & Dreyfus, 1996).

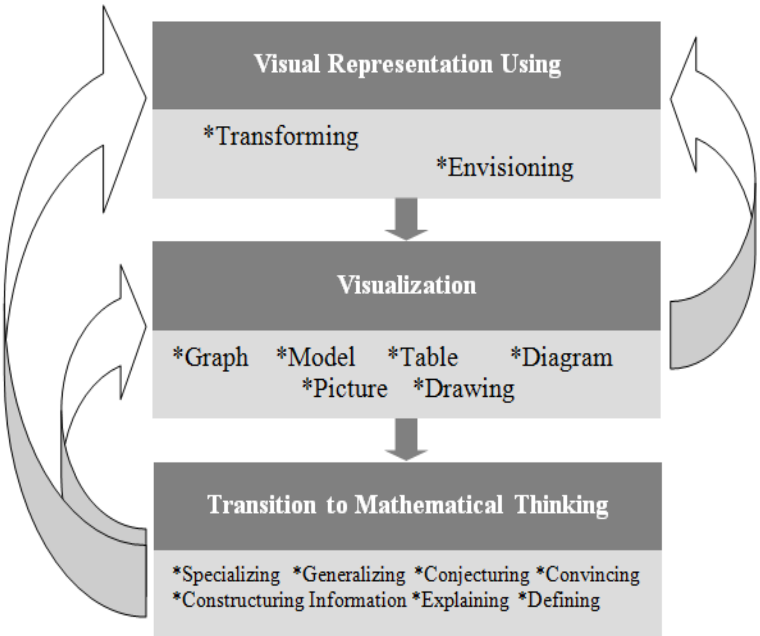


Figure 1. Conceptual Framework for Visual Reasoning (Geçici & Türnüklü, 2021)

Figure 1 shows the conceptual framework of visual reasoning by Geçici & Türnüklü (2021). In this picture, the relationship between aspects related to visual reasoning, especially those related to mathematics, is visible. In connection with the visual reasoning process, the following indicators are Hamid & Idris (2014) detail several indicators of visual reasoning which can be seen in Table 1.

Table 1. Indicators of the Visual Reasoning Process

No	Visual Reasoning Process	Operational Indicators
1	Visual Representation	a. Representing data or information from a diagram, graphic, or table representation. b. Create images of geometric patterns. c. Create drawings to clarify problems and facilitate their resolution
2	Visualization	a. Imagine and visualize concepts in higher dimensions, such as three-dimensional space or more. b. Understand the information provided in a visual format. c. Able to draw visual representations to illustrate concepts, such as drawing number line drawings, or geometric drawings.
3	Transition to mathematical thinking	a. Reducing information into a more general or abstract form. b. Able to formulate problems in mathematical terms, c. Search for the right solution by applying relevant mathematical concepts. d. Able to use mathematical notation correctly and consistently, including symbols, variables, operations, and mathematical expressions.

Based on the results of a preliminary study conducted previously, researchers expected that this difference was related to differences in the abilities of male and female students (Krutetskii 1976) explained that men and women in learning mathematics have differences where men are superior in reasoning, mathematical abilities, and mechanics. Meanwhile, women are superior in accuracy, thoroughness, and thoroughness of thinking.

According to the American Psychological Association (Science Daily, January 6, 2010) (Utami & Rosyidi M.Pd 2016) explained that from the latest international research that men's abilities are better than women, even though men have more self-confidence than women in mathematics, and women from countries where gender equality has been recognized show better abilities in mathematics tests.

Based on the results of the research above showed that there are differences in mathematics learning in terms of gender roles. Therefore, this research links students' visual reasoning based on gender.

METHOD

This research used a qualitative approach to describe students' visual reasoning abilities in solving integral problems. The research involved three undergraduate students from the Tadris Mathematics Program at UIN Sayyid Ali Rahmatullah Tulungagung, consisting of one man and two women. Data was collected through tests and interviews. The test consists of three essay questions as an assessment instrument, while interviews are conducted to gain an in-depth understanding of visual reasoning data. Details of the test instruments used can be seen in Figure 2.

The data analysis method applied involves a narrative descriptive approach, with a qualitative data analysis process done interactively and continuously until all aspects are studied, thereby avoiding data saturation. The data analysis process in this research includes four main stages, namely data collection, data reduction, data presentation, and drawing conclusions based on research results (Creswell 1998).

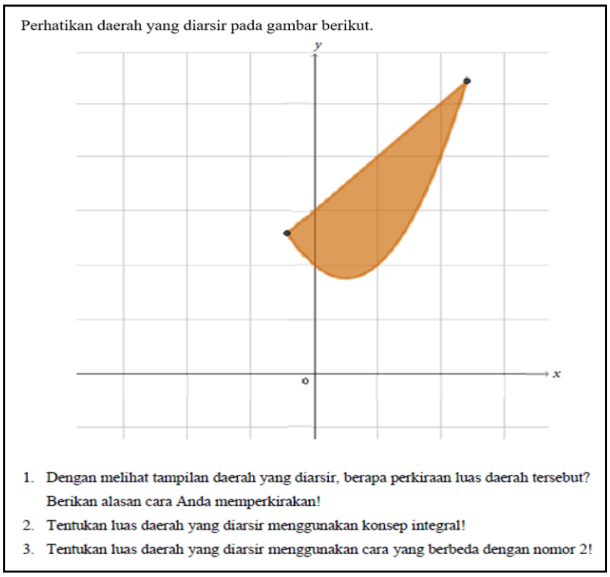


Figure 2. Test instrument

RESULT AND DISCUSSION

Male Students' Visual Reasoning in Solving Integral Problems

Based on Figure 3, students start by redrawing the problem given to determine some important information that can be used to solve the problem. Then proceed by determining the approximate area of the shaded area and testing it using the integral concept. Students first identify the points on the curve and determine the equation of the two curves. By knowing the similarities between the two curves, students begin to determine the intersection point of the two curves to determine the lower and upper limits of the integral as in Figure 4.

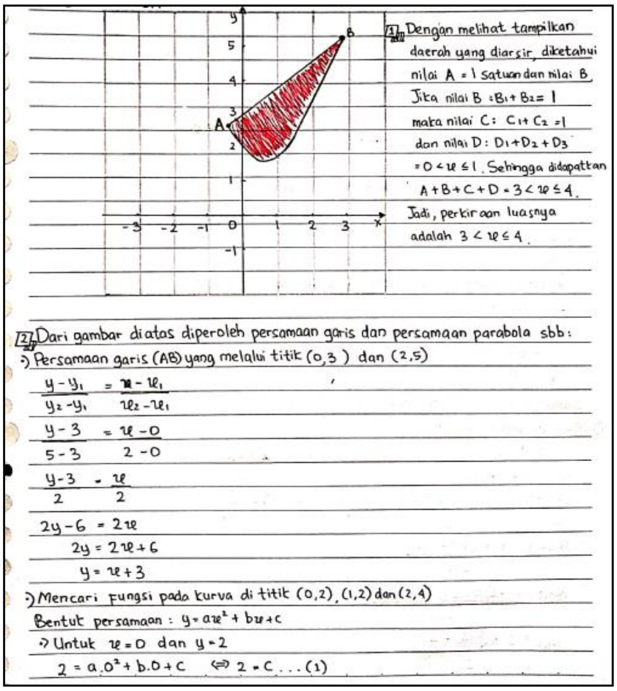


Figure 3. Answer of S1 (a)

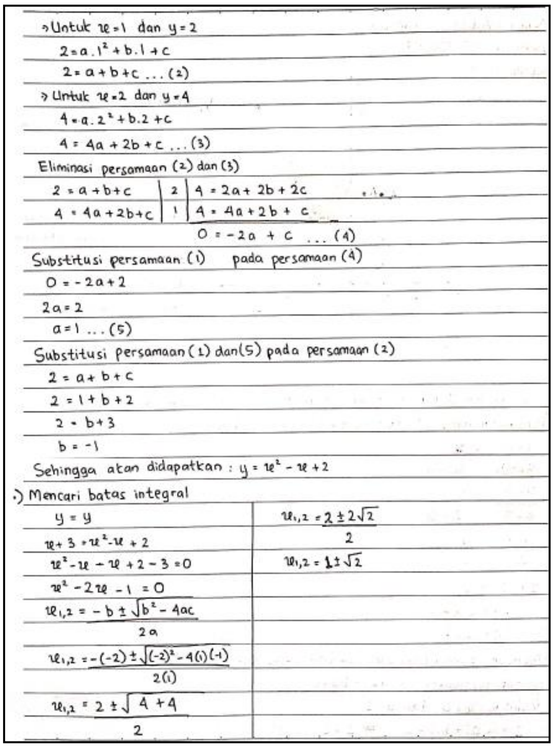
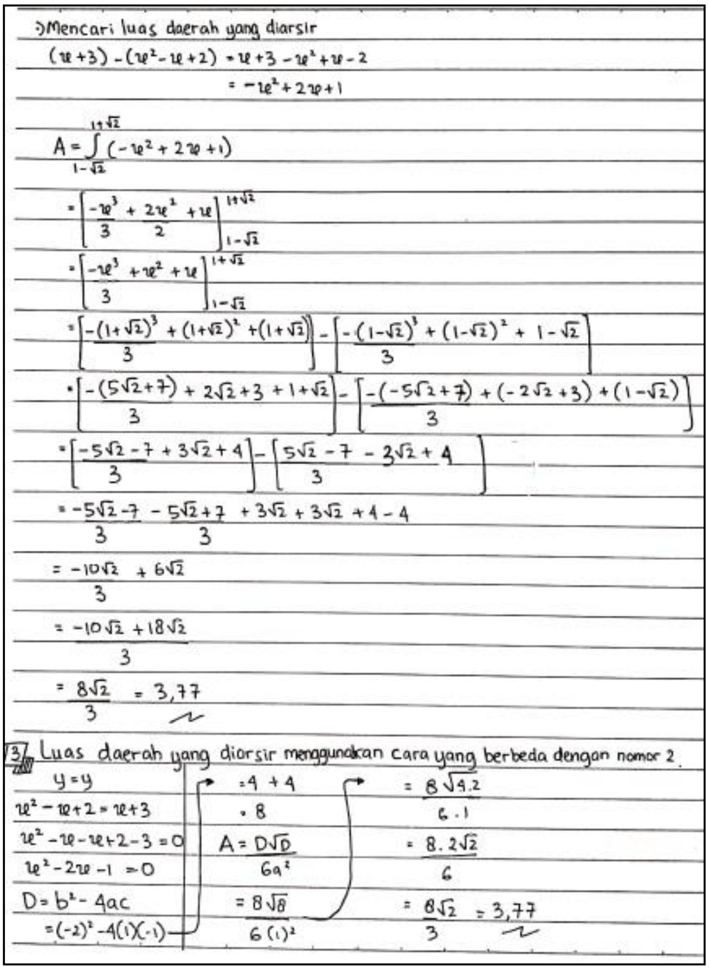


Figure 4. Answer of S1 (b)

Next, determine the area of the shaded area using a definite integral and obtain the result 3.77. Here students do not conclude in terms of units of area. After knowing the area of the shaded area, students try to find the area in different ways. In this different way, students also find the area 3.77. However, the method used is a fast method which is usually often used at school, as in Figure 5.



→ Mencari luas daerah yang diarsir

$(x+3)-(x^2-x+2) = x+3-x^2+x-2$

$= -x^2+2x+1$

$A = \int_{1-\sqrt{2}}^{1+\sqrt{2}} (-x^2+2x+1)$

$= \left[\frac{-x^3}{3} + \frac{2x^2}{2} + x\right]_{1-\sqrt{2}}^{1+\sqrt{2}}$

$= \left[\frac{-x^3}{3} + x^2 + x\right]_{1-\sqrt{2}}^{1+\sqrt{2}}$

$= \left[\frac{-(1+\sqrt{2})^3}{3} + (1+\sqrt{2})^2 + (1+\sqrt{2})\right] - \left[\frac{-(1-\sqrt{2})^3}{3} + (1-\sqrt{2})^2 + 1-\sqrt{2}\right]$

$= \left[\frac{-(5\sqrt{2}+7)}{3} + 2\sqrt{2}+3+1+\sqrt{2}\right] - \left[\frac{-(-5\sqrt{2}+7)}{3} + (-2\sqrt{2}+3) + (1-\sqrt{2})\right]$

$= \left[\frac{-5\sqrt{2}-7}{3} + 3\sqrt{2}+4\right] - \left[\frac{5\sqrt{2}-7}{3} - 3\sqrt{2}+4\right]$

$= \frac{-5\sqrt{2}-7}{3} - \frac{5\sqrt{2}+7}{3} + 3\sqrt{2} + 3\sqrt{2} + 4 - 4$

$= \frac{-10\sqrt{2}}{3} + 6\sqrt{2}$

$= \frac{-10\sqrt{2}+18\sqrt{2}}{3}$

$= \frac{8\sqrt{2}}{3} = 3,77$

3) Luas daerah yang diarsir menggunakan cara yang berbeda dengan nomor 2.

$y = y$

$x^2-x+2 = x+3$

$x^2-x-x+2-3 = 0$

$x^2-2x-1 = 0$

$D = b^2-4ac$

$= (-2)^2-4(1)(-1)$

$= 4+4$

$= 8$

$A = \frac{D\sqrt{D}}{6a^2}$

$= \frac{8\sqrt{8}}{6(1)^2}$

$= \frac{8\sqrt{4.2}}{6.1}$

$= \frac{8.2\sqrt{2}}{6}$

$= \frac{8\sqrt{2}}{3} = 3,77$

Figure 5. Answer of S1 (c)

Undergraduate students can depict and interpret problems or concepts graphically. They can manipulate large areas using graphical representations and can solve problems in ways that involve graphical representations. At this level, their minds tend to be dominated by standard mental images when trying to create graphs to solve problems. In line with (Mehmet, 2021) Increasing students' understanding of concepts and problems can be done by developing visual methods that allow them to see information better. Students can achieve this by intentionally repeating information, thereby increasing retention of the information in their memory.

Female Students' Visual Reasoning in Solving Integral Problems

Based on the illustrations in Figures 6 and Figure 7, it can be seen that students showed various approaches to solving the problems given. From a geometric perspective, they were able to estimate the area of the shaded area, but when tested using integral concepts, they encountered several difficulties. When trying to estimate the area of the shaded area, this student did not redraw the graphic sketch provided. Instead, they presented it in the form of separate symbols and then added them together, producing a final estimate of 4.

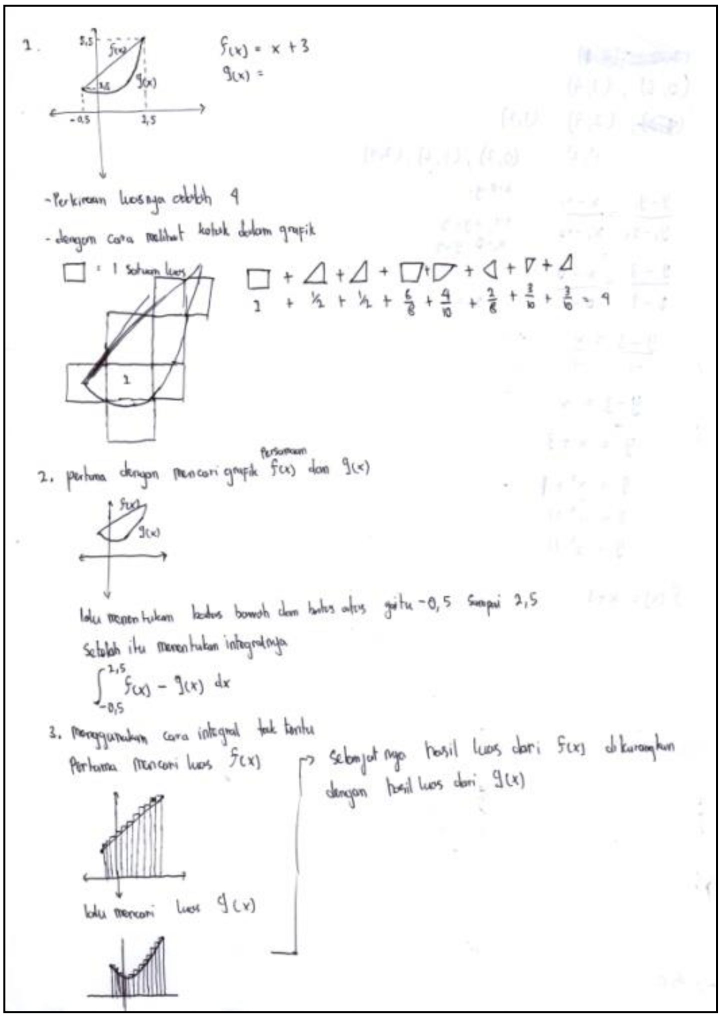


Figure 6. Answer of S2

After trying to test it toward the integral concept, they ran into difficulties. The first step to take was to find the equation of the two curves, but they were unable to determine the equation. The results of the interview after the exam revealed that they had forgotten how to find the equation of a straight line and a parabola, as shown in Figure 3. In addition, regarding other methods for calculating the area of the shaded area, this female student succeeded in meeting the researchers' expectations by determining the area of the area using the approach inner polygon or outer polygon. However, they also failed because they could not find the equation of the given straight line and parabola.

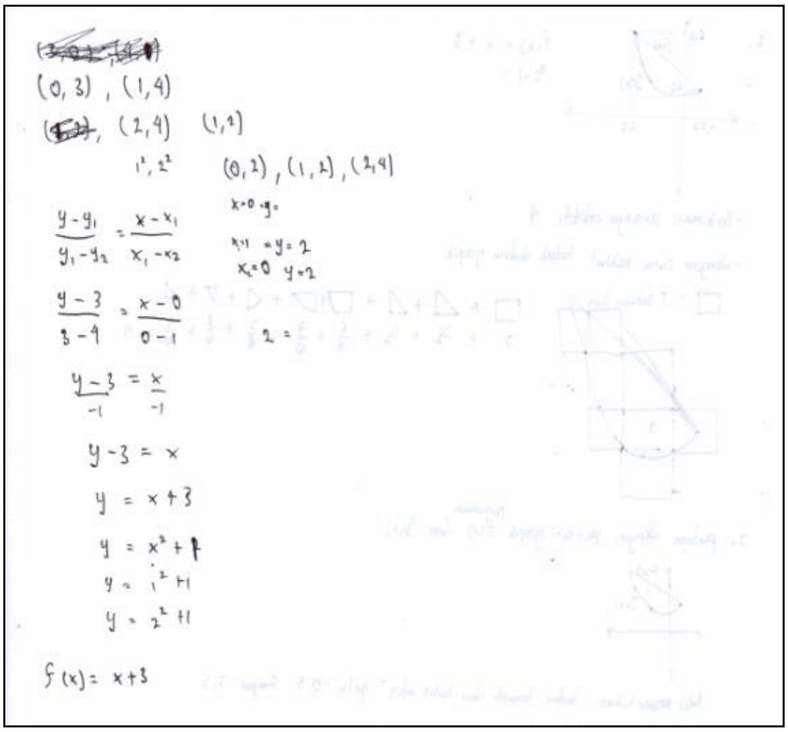


Figure 7. Answer of S3

Observing the way male students solve integrals, it can be seen that they have a good understanding of the concept of visual reasoning. This allows them to represent problems visually, experience a shift in mathematical thinking, and be able to visualize. Assess students' understanding of the concept of visual reasoning, can be done through graphic analysis which focuses on the integration of skills such as reading and interpreting graphs (Ratwani, R. M., Trafton, J. G., & Boehm-Davis 2008). This fits the view Agustin (2016) which stated that to achieve the right problem solution, someone needs to understand and identify the core of the problem.

The results of integral completion for male and female students show that a lack of understanding of the concept of visual reasoning is the main cause. Visual reasoning is described as the ability to effectively use visual elements, such as diagrams or pictures, in doing higher-order thinking tasks (Natsheh, I., & Karsenty 2013). Evaluation of students' visual reasoning can be obtained by utilizing graphics that emphasize combining skills such as reading and interpreting graphic data.

Visual reasoning means having the ability to represent, generate, transform, document, communicate and reflect visual information (Hershkowitz, R., Tabach, M., & Dreyfus 2016). Several students who have good visualization skills can integrate graphical and algebraic representations when solving problems, utilizing their visual skills to visualize complex structures without realizing them.

CONCLUSION

Based on the analysis done, it can be concluded that male students have good abilities in fulfilling the visual representation indicators. They can present information from graphic representations, create geometric patterns, and visualize by understanding information in a visual format. In addition, male students can draw visual representations to illustrate concepts, make the transition to mathematical thinking by reducing information to a more general form, find appropriate solutions by applying mathematical concepts, and use mathematical notation correctly. On the other hand, female students are only limited to the ability to create geometric patterns and solve problems correctly, without showing the same abilities in terms of visual representation and more complex mathematical thinking.

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