Focus ACTion Of Research Mathematic
Volume 9, No. 2, 2026
Page 259-282
https:/jurnalfaktarbiyah.iainkediri.ac.id/index.php/factorm/

ISSN: 2655-3511 (print)
ISSN: 2656-307X (online)

Characteristics of Polya's problem-solving process among senior high students in centers of focus figural pattern generalization

Lazatin 'Aniqoh[1], Sudirman[1], Sisworo[1]

[1]Mathematics Department, The State University of Malang, East Java, Indonesia
***Correspondence**: sudirman.fmipa@um.ac.id

Abstract

Problem-solving ability is a fundamental competency in 21st-century mathematics education. However, 2022 PISA results indicated low performance in Indonesia. This descriptive-qualitative study aimed to describe the characteristics of Polya's problem-solving stages among senior high students in centers-of-focus figural pattern generalization Out of 34 students, three were selected through purposive sampling using Lobato et al.'s three centers of focus: additive growth, relationship between step number and object number, and routine dominated by additive growth. Data were collected through a figural pattern generalization test and semi-structured interviews. Findings showed that students with an additive growth focus solved problems by redrawing patterns, devising a plan based on repeated addition, and carrying out the plan using manual addition without producing general formulas or reviewing answers. The student who focused on the relationship between the step number and the number of objects applied the formula for an arithmetic sequence and verified the resulting solutions. Meanwhile, the student whose focus was on a routine dominated by additive growth transformed the constant increment into a multiplication routine, consistently produced correct answers, and reviewed the solutions. These findings indicate that students' centers of focus in figural pattern generalization are systematically associated with distinct problem-solving strategies at each of Polya's stages, ranging from recursive, additive-based approaches to structural, formula-based reasoning, thereby providing empirical evidence linking Lobato et al.'s center-of-focus framework to Polya's problem-solving stages. In addition, this study offers implications for designing instructional strategies that can support students' transition from recursive to structural reasoning in figural pattern generalization.

Keywords: Pattern Generalization, Polya's Problem Solving, Centers of Focus

How to Cite: 'Aniqoh, L., Sudirman, & Sisworo. Characteristics of Polya's problem-solving process among senior high students in centers of focus figural pattern generalization. *Journal Focus Action of Research Mathematic (Factor M)*, *9*(2), 259–282. https://doi.org/10.30762/f_m.v9i2.8945

INTRODUCTION

Problem-solving ability is one of the most fundamental competencies that students must possess in 21st-century mathematics education (Gunawan et al., 2020; Rahman, 2019; Riyadi et al., 2021). Problem solving is not merely an activity of calculation or memorizing procedures, but rather a complex cognitive process involving systematic observation, critical reasoning, and decision-making to find solutions to previously unfamiliar situations (Rahman, 2019; Szabo et al., 2020). In the context of mathematics education, this ability serves as the foundation for various higher-order thinking skills, including algebraic thinking, logical reasoning, and mathematical generalization (Kieran, 2022). Therefore, it is not surprising that mathematical problem solving has been established as one of the Mathematics Learning Outcomes for secondary education in the Indonesian curriculum (Kemendikbudristek, 2024), reflecting the importance of this competency in preparing students to face real-life challenges (Gunawan et al., 2020).

Nevertheless, empirical evidence indicates that Indonesian students' mathematical problem-solving ability remains low (Lestari et al., 2025). Based on the results of the Program for International Student Assessment (PISA) 2022, Indonesia ranked 68th out of 81 participating countries, with an average mathematics score of 366 points, representing a 13-point decline from the 2018 PISA results (OECD, 2023). Even more concerning, almost no Indonesian students reached Level 5 or 6 in the PISA mathematics assessment, levels at which students are expected to construct complex mathematical models and select and evaluate appropriate problem-solving strategies (OECD, 2023). The weak state of Indonesian students' mathematical problem-solving ability is also reflected in the 2025 Academic Ability Test (*Tes Kemampuan Akademik*/TKA) data, which show that Indonesian senior high school students achieved an average score of 36 out of 100 in mathematics (Kemendikdasmen, 2025). This decline is further corroborated from practitioners' perspective: in interviews with Indonesian mathematics teachers, the drop in PISA 2022 mathematics scores relative to 2018 and 2015 was attributed to six interrelated factors, namely pandemic-related learning loss, curriculum issues, individual student factors, limited educational resources, other student-related factors, and insufficient parental involvement (Wijaya et al., 2024).

Taken together, both data sources (at the international and national levels) converge on the same conclusion: Indonesian students' difficulties lie not in routine computation, but in the higher-order reasoning that constitutes the core of problem-solving ability (Ismeirita et al., 2025; Kania et al., 2024; Masito et al., 2024; Tanudjaya & Doorman, 2020). This condition indicates the need for research that not only evaluates the outcomes of problem solving, but also examines the underlying thinking processes students engage in while solving problems.

One of the most widely used and empirically validated problem-solving frameworks in mathematics education research is the framework proposed by Polya (1973). This framework consists of four interconnected stages: understanding the problem, devising a plan, carrying out the plan, and looking back. Each stage plays a crucial role in determining the quality of students' problem-solving processes and outcomes. This framework has been employed in numerous studies from various angles, including students' problem-solving profiles (Machdalena et al., 2022; Riyadi et al., 2021; Wulan & Anggraini, 2019), trends and development of mathematical problem solving (Toh et al., 2024), students' problem-solving ability (Dewi & Zuroidah, 2023; Hidayah & Murti, 2025), Development of Mathematics Problem-Solving Questions (Domu et al., 2022), and the causes underlying shifts in students' positioning as expert, facilitator, or novice during group problem-solving discussions (Muslim et al., 2024). These findings suggest that the explicit implementation of Polya's framework in instruction can help identify more precisely the stages at which students encounter difficulties, allowing instructional interventions to be designed in a more targeted and effective manner (Gulam & Arenas, 2024; Nico Pradana, 2024). Nevertheless, most of these studies remain focused on categorizing students' levels of mastery at each stage—whether they succeed or fail—without explaining the mathematical aspects that become the center of students' attention as they progress through each stage.

This question becomes especially relevant in the context of figural pattern generalization. Figural pattern generalization is consistently regarded as an important context for developing algebraic thinking, as it requires students to identify structural relationships within visually growing patterns and subsequently express them as general rules (Afkhami et al., 2023; Mora et al., 2024; Wilkie, 2022). Thus, pattern generalization serves as an important bridge from concrete arithmetic thinking to abstract algebraic

http://doi.org/10.30762/f_m.v9i2.8945

thinking and represents one of the most productive early pathways for students to develop an understanding of functional relationships (El Mouhayar, 2018; Wilkie, 2022).

One concept that plays a key role in this generalization process is "centers of focus," a concept introduced by Lobato, Hohensee, and Rhodehamel (2013). Centers of focus are properties, features, regularities, or conceptual objects that a student notices, thereby influencing the direction and quality of the mathematical reasoning they develop (Kularajan et al., 2026; Risit et al., 2025; Wilkie, 2024). In the context of visually growing figural patterns, Lobato et al. (2013) identified that students may focus on additive growth from one figure to the next, on the relationship between step number and number of objects, or on a routine dominated by additive growth.

Nevertheless, research on problem solving and research on focus have largely developed along separate tracks. Studies employing Polya's framework have primarily focused on identifying which problem-solving stages students succeed in or fail to master (Hidayah & Murti, 2025), without examining the mathematical aspects that students focus on at each stage. In contrast, several studies on centers of focus have examined how students' attention develops while carrying out pattern generalization tasks (Lobato et al., 2012; Wilkie, 2022) and mathematical modeling (Kularajan et al., 2026), yet have not situated this focus within a systematic problem-solving process framework. Consequently, although each perspective has explained part of the source of students' difficulties, the two have not been integrated to explain one another. It thus remains unknown how a particular center of focus manifests at each stage of problem-solving within Polya's framework. For instance, it is unclear whether students who focus on the relationship between the number of steps and the number of objects understand the problem differently from students who focus on additive growth, or whether a particular focus is more closely associated with students' success in devising and carrying out a solution plan.

Linking these two perspectives allows the scope of both to be extended. For the concept of student noticing, this study explains not only what constitutes students' centers of focus, but also how this attention is enacted in problem-solving behavior at each structured stage. For Polya's framework, this study offers a more detailed account of why students succeed or encounter difficulty at each stage by examining the mathematical characteristics that underlie their reasoning rather than merely their level

 http://doi.org/10.30762/f_m.v9i2.8945

of mastery. Theoretically, this study integrates the concept of centers of focus with Polya's problem-solving framework into a single analytical perspective to explain how students' mathematical attention shapes the characteristics of their problem-solving processes in the context of figural pattern generalization. In practice, the findings of this study are expected to help teachers identify students' focal areas during learning, thereby enabling them to design more appropriate instructional strategies to support students' problem-solving abilities and algebraic thinking.

Based on this research gap, the research question guiding this study is: What are the characteristics of senior high school students' problem-solving in solving figural pattern generalization problems based on their centers of focus? To answer this question, this study examines the problem-solving processes of senior high school students with different centers of focus as they carry out figural pattern generalization tasks. By integrating the perspective of centers of focus with Polya's problem-solving framework, this study is expected to provide a more comprehensive understanding of how students' mathematical attention shapes the characteristics of their problem-solving processes. The findings of this study are expected not only to enrich the theoretical understanding of the relationship between centers of focus and problem-solving stages, but also to provide a foundation for developing instructional strategies that facilitate students' transition from recursive to structural reasoning in mathematics learning.

METHODS

This study was situated within an interpretive (constructivist) qualitative paradigm, which holds that knowledge about students' problem-solving processes is not an objective fact that exists independently of the researcher, but rather a meaning that is jointly constructed through the researcher's interaction with, and interpretation of, students' verbal and written expressions. Consistent with this epistemological stance, the researcher functioned as the primary instrument of data collection and interpretation, actively probing students' reasoning during the interviews and interpreting their written work in light of the interview data, rather than assuming a neutral, detached position from the phenomenon under study (Creswell & Poth, 2018; Guba & Lincoln, 1994). This study employed a descriptive qualitative research design to obtain an in-depth and accurate understanding of students' problem-solving performance characteristics at each

 http://doi.org/10.30762/f_m.v9i2.8945

stage of Polya's framework, based on their respective tendencies in centers of focus. A qualitative descriptive approach is particularly suitable for exploring complex, context-specific educational phenomena that are not yet fully theorized or categorized (Creswell & Creswell, 2023; Sandelowski, 2000).

Therefore, a qualitative descriptive design allows the researcher to describe variations in performance at each stage of Polya's framework based on actual student performance, interview data, and problem-solving processes. This approach supports the inductive development of descriptions grounded in students' authentic experiences in solving figural pattern generalization problems.

Research Subjects

Based on this analysis, students were classified into three categories in Table 1. From each category, one information-rich participant was then selected through purposive sampling (Miles et al., 2014). Purposive sampling is appropriate in qualitative research when the goal is to obtain an in-depth understanding from information-rich cases rather than to generalize findings across a population. The selection of these three subjects was based on several criteria. First, the student demonstrated a consistent focus across all administered tasks. Second, the student completed all tasks and produced sufficiently complete written responses to allow the researcher to interpret their reasoning process. Third, the student clearly articulated their thinking during the semi-structured interview. Fourth, the explanations given during the interview were consistent with their written responses.

Table 1. Centers of Focus in Linear Figural Pattern Generalization

Center of Focus	Description
Additive growth	Students pay attention to changes in the number of objects in consecutive figures within a visually growing pattern.
Relationship between step number and number of objects	Students recognize the relationship between the step number and the number of objects in the corresponding figure.
Routine dominated by additive growth	Students focus again on additive growth. In this case, students connect it to a routine for determining the number of objects in the targeted figure.

(Lobato et al., 2013; Wilkie, 2024)

This study does not aim to compare variation among students within the same category of center of focus, but rather to obtain an in-depth understanding of how each theoretically established center of focus manifests at each stage of Polya's problem-

 http://doi.org/10.30762/f_m.v9i2.8945

solving framework. Therefore, one information-rich participant per category was considered sufficient to address the research objective, consistent with the qualitative descriptive design's emphasis on depth of understanding over breadth of representation (Sandelowski, 2000).

Research Procedure

The research procedure was carried out through the following stages. Preparation Stage: developing the research instruments in the form of a figural pattern generalization test and interview guidelines, as well as validating the instruments by experts before use. Implementation Stage: administering the test to 34 students to identify each student's center of focus. The test results were analyzed to classify students into 3 categories of centers of focus based on Lobato's Center of Focus. Data Processing and Analysis Stage: selecting three students representing the categories of centers of focus to participate in in-depth interviews aimed at exploring their problem-solving processes based on Polya's stages. The in-depth interviews lasted approximately 30 minutes and were recorded using a voice recorder. Report Writing Stage: preparing the research report based on the completed data analysis.

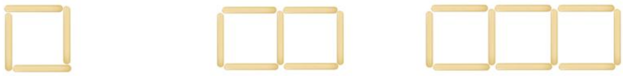
Research Instrument

The instruments used in this study consisted of two types: test instruments and interview guidelines. The test instruments consisted of three figural pattern generalization problems designed to measure students' abilities to understand, plan, carry out, and review solutions to pattern generalization problems. The three problems were designed with varying levels of difficulty and presented as figural representations to more clearly identify students' centers of focus. These three problems are presented in Table 2.

Table 2. Pattern Generalization Test Instrument

No.	Problem	
1	Vernon enjoys arranging toothpicks into various unique shapes. One day, he created a sequence of patterns that changed slightly at each step. He then challenged his friends to determine the rule governing the changes from the first, second, third, and subsequent patterns as illustrated below. 	Based on the pattern shown: **a.** How many toothpicks does Vernon need for the 5^{th} shape? Explain how you obtained your answer! **b.** How many toothpicks does Vernon need for the 15^{th} shape? Explain how you obtained your answer!

http://doi.org/10.30762/f_m.v9i2.8945

	Shape 1 Shape 2 Shape 3	**c.** How many toothpicks does Vernon need for the **n** shape, where **n** is any natural number? Explain how you obtained your answer!
2	At a playground, Hoshi found many used popsicle sticks that were still clean and intact. She then began arranging them as shown in the pattern below.  Shape 1 Shape 2 Shape 3	Based on the pattern shown: **a.** How many popsicle sticks does Hoshi need for the 12th shape? Explain how you obtained your answer! **b.** How many popsicle sticks does Hoshi need for the 25th shape? Explain how you obtained your answer! **c.** How many popsicle sticks does Hoshi need for the n shape, where n is any natural number? Explain how you obtained your answer!
3	Woozi enjoys building miniature houses using straws. He started with one simple house and then tried connecting it to form two houses standing side by side. As he built more, the arrangement increasingly resembled a row of houses in a small village, as shown in the pattern below.  Shape 1 Shape 2 Shape 3	Based on the pattern shown: **a.** How many straws does Woozi need for the 35th shape? Explain how you obtained your answer! **b.** How many straws does Woozi need for the 48th shape? Explain how you obtained your answer! **c.** How many straws does Woozi need for the n shape, where n is any natural number? Explain how you obtained your answer!

The interview guidelines were developed in a semi-structured format to explore students' thinking processes in depth at each stage of Polya's problem-solving framework. Before use, both instruments were validated by three lecturers from Universitas Negeri Malang, who served as expert validators to ensure their appropriateness and alignment with the study's objectives. The validation was conducted using a checklist-based validation sheet covering four aspects: (1) the alignment of the instrument's content with the research objectives, (2) the accuracy of the mathematical concepts being measured, (3) the clarity of the language and task instructions, and (4) the suitability of the interview questions for eliciting students' thinking processes. For each aspect, the validators assessed whether the instrument met the expected criteria and provided comments and suggestions regarding several ambiguous terms. Overall, all three validators agreed that the instruments were appropriate for use in this study. Based on their feedback, minor revisions were made to refine the wording of several test items, task instructions, and interview questions to improve clarity and reduce the likelihood of differing interpretations.

Data Analysis Technique

Data analysis followed the interactive qualitative analysis procedure of Miles, Huberman, and Saldaña (2014), comprising data condensation, data display, and conclusion drawing/verification, combined with deductive and inductive coding. In the data condensation stage, students' written responses and interview transcripts were first coded deductively using the three centers of focus (Lobato et al., 2013) as a priori codes to confirm each subject's dominant focus category. Within each focus category, the data were then coded deductively using Polya's (1973) four problem-solving stages as a second layer of a priori codes, complemented by inductive coding to capture emergent characteristics of how that particular focus manifested at every stage. These codes were iteratively compared within and across the three subjects (constant comparison) to develop descriptive themes that explain how each center of focus shaped students' reasoning at each stage of problem solving, rather than merely classifying students as successful or unsuccessful. In the data display stage, coded excerpts were organized into a matrix cross-tabulating centers of focus and Polya's stages (Table 3) to facilitate systematic comparison. Finally, in the conclusion-drawing and verification stage, a narrative description of each subject's characteristics was constructed from the resulting themes and checked against the raw data to ensure that the interpretation remained faithful to the subjects' spoken and written responses.

Table 3. Polya's Problem-Solving Stages

Stage	Indicator
Understanding the problem	Students carefully consider the parts of the problem from various perspectives attentively rather than merely reading the problem passively.
Devising a plan	Students can identify the plan to be used in solving the problem.
Carrying out the plan	Students implement the developed plan.
Looking back	Students reflect on or verify the obtained solution.

(Polya, 1973)

RESULT AND DISCUSSION

This study examined the characteristics of senior high school students' problem-solving stages when solving figural pattern generalization problems, based on Polya's

 http://doi.org/10.30762/f_m.v9i2.8945

framework, in terms of the centers of focus they employed. Of the 34 students who participated in the preliminary test, three were selected as research subjects, each representing a category of focus based on the framework proposed by Lobato et al. (2013). Table 4 presents the number of students in each center-of-focus category identified from their responses.

Table 4. Results of Student Response Identification Based on Centers of Focus

Center of Focus	Number of Students
Additive Growth	5
Relationship Between Step Number and Number of Objects	13
Routine Dominated by Additive Growth	11
Others	5

Data analysis was conducted through three stages of coding. First, interview transcripts and written response sheets were coded using the indicators in Table 3 as an initial coding scheme to identify data units relevant to each stage of Polya's model (understanding the problem, devising a plan, carrying out the plan, and looking back). Second, the coded data units were grouped according to the "center of focus" categories in Table 1, so that each quote had two labels: the Polya stage and the focus category. The coding of the specific research subjects' focus categories is presented in Table 5. Third, the codes were synthesized into descriptions of each subject's characteristics at each stage, which were then cross-checked against the raw data (answer sheets and transcripts) to ensure the reliability of the interpretations.

Table 5. Coding of Research Subjects Based on Centers of Focus

Subject	Focus Category
S1	Focus on Additive Growth
S2	Focus on the Relationship Between Step Number and Number of Objects
S3	Focus on a Routine Dominated by Additive Growth

Cross-Subject General Patterns

Before elaborating in detail on the characteristics of each subject, the cross-subject analysis reveals three general patterns. First, the quality of problem comprehension and planning increased consistently as the focus shifted from additive growth (S1), to routine dominated by additive growth (S3), and finally to the relationship between step number and number of objects (S2). Second, a parallel pattern emerged at the looking back stage: the intensity and quality of verification increased in line with the level of abstraction of students' focus, ranging from the absence of verification (S1), to verification based on matching computational results with the shape (S3), to verification based on testing the validity of the formula across different values of n (S2). Third, all three subjects demonstrated that the representation constructed during the understanding-the-problem stage tended to be sustained through subsequent stages: the visual-local representation in S1 remained recursive through to the final stage. In contrast, the structural representation in S2 remained symbolic through to the verification stage.

These patterns indicate that center of focus does not merely influence the outcome of pattern generalization, as reported by Lobato et al. (2013), but functions as a consistent determinant throughout the entire Polya problem-solving process, from how the problem is initially interpreted to how the solution is ultimately verified. The detailed characteristics of each subject at each stage are presented in Table 6 and are further elaborated in the following section.

Table 6. Synthesis of the Three Subjects' Characteristics at Each Stage of Polya's Problem-Solving Process

Polya's Stage	**S1 - Additive Growth**	**S2 - Relationship Between Step Number and Number of Objects**	**S3 – Routine Dominated by Additive Growth**
Understanding the Problem	Fails to construct meaning of the problem; redraws the pattern sequentially; attention centered on visual changes across figures.	Interprets the pattern as an arithmetic sequence from the outset; without redrawing, moves directly toward the relationship between figure (pattern) number and object quantity.	Identifies the regular increase in objects; begins searching for the underlying pattern. $(2 \rightarrow 4)$.
Devising a Plan	Repeated addition as the sole strategy; no alternatives considered; does not move toward a general formula.	The arithmetic sequence formula $U_n = a + (n-1)b$ is consciously selected as a symbolic representation of the variable relationship.	Additive growth is converted into a multiplicative routine, $1 + (b \times n)$becomes the self-constructed "anchor point" of the formula.
Carrying Out the Plan	Effective for small n; computational errors emerge for large n; does not produce a general formula.	Systematic substitution simplified into a linear form (e.g., $2n + 1$) reflects algebraic thinking (Kieran, 2022).	The self-constructed formula is applied consistently; results are checked against a specific figure; and a correct

			generalization is produced.
Looking Back	No verification activity; the process is considered complete once the repeated addition ends.	Tests the formula with values of n different from those given in the item; verification oriented toward the validity of the mathematical relationship.	Test the formula by matching computed results to a specific figure; verify visually.

Characteristics of Each Subject

Focus on Additive Growth (S1)

Students with a focus on additive growth attend to changes in the number of objects between consecutive shapes without explicitly recognizing the relationship between the number of shapes and the magnitude of growth (Lobato et al., 2013). This characteristic appeared consistently across all four problem-solving stages for S1 (Table 6).

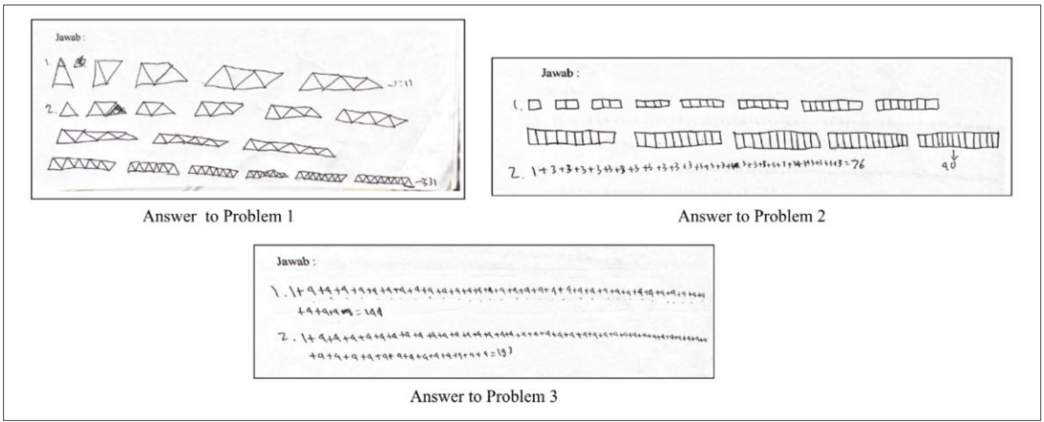


Figure 1. S1's Answer Sheet

Understanding the Problem

S1 was unable to restate the problem in his own words and immediately redrew the pattern sequentially (Figure 1). This response was coded as a failure to construct meaning of the problem: S1's focus on visual changes between figures diverted attention from the underlying mathematical relationship in the pattern.

Devising a Plan

S1 stated repeated addition as the only strategy that came to mind: "*because I was asked to find the fifth shape, so I drew up to that shape*", without considering any alternatives. This focus on local growth between shapes constrained the planning process, such that a functional relationship between variables never formed.

Carrying Out the Plan

This strategy remained accurate for small values of n but produced computational errors as n increased. S1 conceived of growth as a process of stepwise addition "*add 3 more until reaching 76*", rather than as a functional relationship between shape number and object quantity, consistent with the findings of Barahmand and Attari (2025) that reasoning based solely on increment does not support the formation of structural generalization.

Looking Back

No verification activity occurred. S1 confirmed that checking was not performed because a general formula had not yet been identified, underscoring that in the absence of a general representation, the need to reflect on the accuracy of the answer did not arise.

Focus on the Relationship Between Step Number and Number of Objects (S2)

S2 was a subject focused on the relationship between step number and the number of objects. The following presents the analysis of S2's problem-solving process in solving the three figural pattern generalization problems based on Polya's stages, as shown in Figure 2.

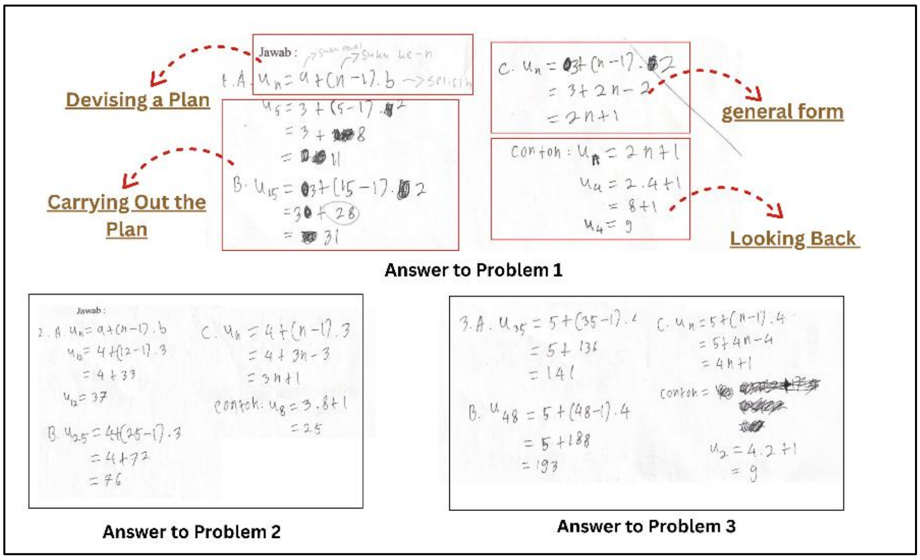


Figure 2. S2's Answer Sheet

Understanding the Problem

S2 immediately interpreted the pattern as an arithmetic sequence and directed attention toward determining the number of objects in the n^{th} shape, without redrawing the pattern. This response was coded as identification of the structural relationship from the initial stage onward.

Devising a Plan

S2 consciously selected the arithmetic sequence formula $Un = a + (n-1)b$ as a symbolic representation of the relationship between variables, consistent with what Wilkie (2016) describes as the ability to articulate covariational relationships directly, an indicator of genuine functional understanding.

Carrying Out the Plan

S2 substituted values systematically and simplified the results into linear forms $(2n+1, 3n+1, 4n+1)$ demonstrating algebraic manipulation that reflects a level of algebraic thinking as described by Kieran (2022).

Looking Back

S2 consistently tested the simplified formula using values of n different from those given in each item, indicating an orientation toward the correctness of the constructed mathematical relationship, rather than merely the correctness of the answer.

Focus on a Routine Dominated by Additive Growth

S3 was a subject focused on a routine dominated by additive growth. The following presents the analysis of S3's problem-solving process for the three figural pattern generalization problems, based on Polya's stages shown in Figure 3.

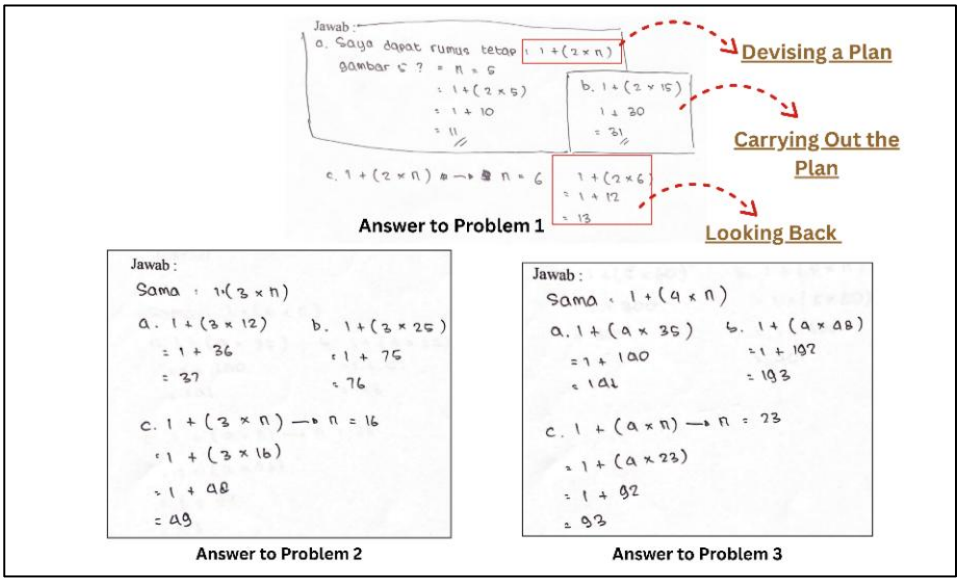


Figure 3. S3's Answer Sheet

Understanding the Problem

S3 identified the regular increase in objects but, at one point, questioned why the magnitude of the increase changed: *"from shape 1 to shape 2 it increases by two, but from the third shape it increases by four".* S3's attention did not remain fixed on visual changes as with S1, but began searching for the underlying regularity; S3's focus began with additive growth yet shifted toward a more structured relationship.

Devising a Plan

S3 transformed the observed increment into a multiplicative routine, $1 + (b \times n)$, treating the initial increment as the "*anchor point*" of the formula. This routine was not constructed through a formal relationship as in S2, but yielded a more efficient representation than recursive addition.

Carrying Out the Plan

The self-constructed formula was applied consistently across all three items, producing correct generalizations $(1 + 2n, 1 + 3n, 1 + 4n)$ without reliance on repeated addition.

Looking Back

S3 tested the formula by matching computed results against a specific shape. It concluded that the results were "*consistent*", a form of verification grounded in visual representation, distinct from S2's symbolic verification, yet still indicative of reflective awareness of the correctness of the result.

Discussion

Why Center of Focus Shapes Distinct Stage Characteristics

The differences in characteristics across the four Polya stages among S1, S2, and S3 can be explained by the following cognitive mechanism: students' focus at the understanding-the-problem stage determines which mathematical object the student represents, and this representation is subsequently carried consistently into the following stages. When a student's focus is limited to visual changes between shapes (S1), the representation formed is local and event-based, step by step, such that the most "natural" strategy selected is repeated addition, not because the student is incapable of calculating, but because the pattern's structure as a whole has never been represented as a cognitive object. This is consistent with Radford (2010), who asserts that meaningful

http://doi.org/10.30762/f_m.v9i2.8945

generalization requires the ability to perceive the invariant structure underlying visual change, rather than merely counting the change itself. This characterization aligns closely with Firnanda and Susiswo (2024), who found that senior high school students solving figural number-pattern problems likewise produced errors when they drew figures spontaneously without a ruler and relied on memorized arithmetic facts.

Conversely, when a student's focus is directed from the outset toward the relationship between step number and the number of objects (S2), the resulting representation is covariational, and the two variables are understood as functionally interrelated (Wilkie, 2022, 2024). It is this covariational representation that enables S2 to select the arithmetic sequence formula directly as a plan, to manipulate it algebraically during the execution stage (reflecting algebraic thinking, Kieran, 2022), and to retest it during the verification stage, because the symbolic representation provides an evaluative tool that S1 does not possess. In other words, the quality of verification at the looking-back stage is not merely a matter of a student's habit of rechecking, but a direct consequence of whether a testable general representation is available.

S3's characteristics reveal that the relationship between focus and cognitive representation is gradual rather than categorical. S3 likewise originates from additive growth (as does S1). However, it organizes this growth into a multiplicative routine that operates like a formula, even though it is not derived from a formal relationship between variables. This finding provides empirical support for Lobato et al.'s (2013) description of the additive growth based routine focus as a distinct category, while also extending it by showing that this category functions cognitively as a bridge: the routine S3 constructed enables efficient execution of the plan and verification based on representational matching (two characteristics that S1 does not possess) even though S3's manner of interpreting the problem at the initial stage remains rooted in additive growth.

Conceptual Model: Integration of Center of Focus and Polya's Stages

Based on the patterns described above, this study proposes a conceptual model explaining how center of focus functions as a cognitive entry point that determines the student's representation at the understanding-the-problem stage, which is then carried consistently through the stages of devising a plan, carrying out the plan, and looking back. The flow of this model is presented in Table 7.

Table 7. Conceptual Model of the Influence Pathway from Center of Focus to the Stages of Polya's Problem-Solving Process

Center of Focus (entry point)	Cognitive Representation Formed	Consequences for Devising & Carrying Out the Plan	Consequences for Looking Back
Additive Growth (S1)	Local/perceptual representation based on visual change between shapes.	Recursive strategy (repeated addition); no general formula; prone to error at large n manipulation toward a linear form.	No general evaluative tool → no verification.
Relationship Between Step Number and Number of Objects (S2)	Covariational/structural representation between variables.	Explicit formula (arithmetic sequence); algebraic manipulation toward a linear form.	Verification based on representational matching (visual).
Routine Dominated by Additive Growth (S3)	Transitional representation: additive growth organized into a repeated routine.	Self-constructed routine (e.g., $1 + (b \times n)$); efficient but not yet based on a formal relationship.	Verification based on testing the formula's validity at new values of n.

This model extends the concept of focusing phenomena from Lobato et al. (2013), which originally explained how focus influences the quality of pattern generalization, by demonstrating that this influence is cross-stage and cumulative throughout the problem-solving process, rather than confined to the final generalization product.

Positioning of the Findings Relative to Prior Research

These findings support Radford (2010) and Barahmand and Attari (2025) regarding the limitations of reasoning based solely on increment (the case of S1), as well as Wilkie (2022, 2024) regarding the role of covariational representation in functional understanding (the case of S2). In line with this, the findings are also consistent with Utami et al. (2023), who found that differences in how students construct pattern generalizations originate from differences in their focus when identifying the pattern. A comparable developmental trajectory was documented by Kama et al. (2023) among sixth-grade students, who tended to generalize near terms arithmetically before attempting a general rule, but frequently skipped genuine far-term (algebraic) generalization and instead calculated far terms by rote once a rule was "found." This pattern parallels the present study's S1, whose additive, near-term reasoning never developed into a functional rule, and stands in contrast to S2, whose covariational focus supported a complete transition to algebraic generalization—reinforcing the argument that the presence or absence of a structural center of focus, rather than arithmetic competence alone, differentiates students who complete this transition from those who do not.

At the same time, this study extends the existing literature in two respects. First, prior studies employing the Polya framework (Hidayah & Murti, 2025; Machdalena et al., 2022; Riyadi et al., 2021) have generally reported students' levels of mastery at each stage without explaining the underlying mathematical aspects; this study extends that framework by showing that the center of focus is one cognitive factor that can explain variation in stage mastery. Second, prior research on focusing phenomena (Lobato et al., 2013; Wilkie, 2022) has concentrated on the generalization process itself; this study extends that scope to the entire problem-solving cycle, including devising a plan and looking back stage, which have not previously been explicitly linked to center of focus in the existing literature.

This study did not identify any patterns that contradict these reference theories; the absence of contrasting findings is likely attributable to the scope of subjects being limited to three theoretically established focus categories (Lobato et al., 2013). Further research incorporating greater variation in pattern context or educational level is needed to examine whether the model in Table 6 also holds for other focus categories or under conditions that may potentially yield different patterns.

Theoretical and Practical Implications

Theoretically, this study contributes a model that integrates the concept of center of focus with Polya's problem-solving framework (Table 7), which may serve as a foundation for further research on mathematical noticing in problem-solving contexts more broadly, not only in the context of pattern generalization. This model generates propositions that can be further tested: (1) the cognitive representation formed at the understanding-the-problem stage tends to determine the type of strategy selected at the devising-a-plan stage; (2) the availability of a general representation (a formula or routine) constitutes a prerequisite for the emergence of verification activity at the looking-back stage; and (3) the additive-growth-based routine focus functions as a transitional category that cognitively bridges recursive reasoning and structural reasoning.

 http://doi.org/10.30762/f_m.v9i2.8945

Practically, these findings imply that pattern-generalization instruction needs to be deliberately designed to shift students' focus from local observation (change between shapes) toward recognition of the structural relationship (step number–number of objects), for example, through prompting questions that encourage students to relate the step number to the number of objects from the understanding-the-problem stage onward, rather than allowing students to construct the pattern recursively first. By recognizing students' center of focus early on, educators can provide interventions targeted at the stage posing the greatest risk for each focus category, support for constructing structural meaning for students focused on additive growth, and reinforcement of formal verification habits for students who have already developed an efficient routine but have not yet grounded it in a formal relationship.

CONCLUSION

This study concludes that the center of focus serves as a cognitive entry point that shapes students' problem-solving characteristics across all stages of Polya's framework, extending Lobato et al.'s (2013) notion of focusing phenomena beyond its original association with the outcome of pattern generalization. The representation formed during the understanding the problem stage tends to persist through the looking back stage, indicating that focus operates as a cross-stage, cumulative mechanism. Across the three subjects examined, the student with an additive growth center of focus (S1) constructed a local, visually driven understanding that confined problem-solving to recursive strategies and yielded no general formula or verification. The student with a relationship of step number and number of objects center of focus (S2) established a covariational representation from the outset, enabling the systematic formulation, execution, and verification of an arithmetic-sequence formula. The student with a routine dominated by additive growth (S3) exhibited transitional characteristics, reorganizing additive growth into a more efficient multiplicative routine. However, the resulting formula was not yet grounded in a formal relationship between variables.

These findings yield a conceptual model integrating center of focus with Polya's problem-solving framework, explaining how focus determines the cognitive representation formed at the understanding stage, which in turn shapes the type of strategy devised, the manner of execution, and the availability of evaluative tools at subsequent stages, offering a cognitive explanation not previously articulated within Polya's framework.

Theoretically, this study contributes to research on mathematical noticing by demonstrating that the quality of problem solving is determined not merely by procedural mastery but by the mathematical object that first captures a student's attention. In practice, the findings imply the need for pattern-generalization instruction deliberately designed to shift students' focus from local observation to recognizing structural relationships, accompanied by interventions tailored to the risks associated with each focus category.

This study is limited to three subjects representing focus categories that are already theoretically established (Lobato et al., 2013). Future research is recommended to test the proposed model across a broader range of pattern contexts and educational levels.

ACKNOWLEDGMENTS

The authors would like to thank the expert validators from Universitas Negeri Malang for their valuable input on the research instruments, as well as the school and students who participated in this study.

FUNDING STATEMENT

This work received no specific grant from any public, commercial, or not-for-profit funding agency.

AUTHOR CONTRIBUTIONS

First Author: Conceptualization, methodology, analysis, writing - original draft, editing, and visualization;

Second Author: Review & editing, validation, formal analysis, and supervision;

Third Author: Validation and supervision.

REFERENCES

Afkhami, R., Asghary, N., & Medghalchi, A. (2023). Developing a framework for evaluating student's understanding of figural pattern generalization. *PNA, 18*(1), 57–76. https://doi.org/10.30827/pna.v18i1.16566Ba

Barahmand, A., & Attari, N. (2025). Investigating students' reasoning in generalization of deconstructive figural patterns: Investigating students' reasoning in generalization of deconstructive figural patterns. *Educational Studies in Mathematics, 118*(1), 53–70. https://doi.org/10.1007/s10649-024-10355-z

Creswell, J. W., & Creswell, J. D. (2023). *Research design: Qualitative, quantitative, and mixed methods approaches* (6th ed.). SAGE Publications.

Creswell, J. W., & Poth, C. N. (2018). *Qualitative inquiry and research design: Choosing among five approaches* (4th ed.). SAGE Publications.

Dewi, P. M., & Zuroidah, N. (2023). Analisis kemampuan pemecahan masalah matematis siswa kelas XI MAN 1 Kota Kediri dengan langkah IDEAL ditinjau dari kemandirian belajar. *Journal Focus Action of Research Mathematic (Factor M), 6*(1), 153–172. https://doi.org/10.30762/f m.v6i1.1246

Domu, I., Manangin, S. A., & Pinontoan, K. F. (2022). Pengembangan soal pemecahan masalah untuk siswa sekolah menengah pertama menggunakan pendekatan pendidikan matematika realistik indonesia. *Journal Focus Action of Research Mathematic (Factor M), 5*(1), 1–13. https://doi.org/10.30762/f m.v5i1.605

El Mouhayar, R. (2018). Trends of progression of student level of reasoning and generalization in numerical and figural reasoning approaches in pattern generalization. *Educational Studies in Mathematics, 99*(1), 89–107. https://doi.org/10.1007/s10649-018-9821-8

Firnanda, G. I., & Susiswo. (2024). Students' dual processes in solving number pattern questions and providing scaffolding. *Journal Focus Action of Research Mathematic (Factor M), 7*(2). https://doi.org/10.30762/f m.v7i2.3449

Guba, E. G., & Lincoln, Y. S. (1994). Competing paradigms in qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 105–116). SAGE Publications.

Gulam, A. J. B., & Arenas, J. C. (2024). Mathematics performance and polya's method in problem solving. *World Journal of Advanced Research and Reviews, 23*(3), 2156–2162. https://doi.org/10.30574/wjarr.2024.23.3.2873

Gunawan, G., Harjono, A., Nisyah, M., Kusdiastuti, M., & Herayanti, L. (2020). Improving students' problem-solving skills using inquiry learning model combined with advance organizer. *International Journal of Instruction, 13*(4), 427–442. https://doi.org/10.29333/iji.2020.13427a

Hidayah, R. N., & Murti, R. C. (2025). Evaluating mathematical problem-solving skills in primary school students: Challenges and insights from Polya's framework. *AL-ISHLAH: Jurnal Pendidikan, 17*(3). https://doi.org/10.35445/alishlah.v17i3.6534

Ismeirita, I., Ahman, E., Dahlan, D., & Supendra, D. (2025). Identifying key factors influencing the development of higher order thinking skills (hots) in students: A systematic literature review. *AL-ISHLAH: Jurnal Pendidikan, 17*(3). https://doi.org/10.35445/alishlah.v17i3.6627

Kama, Z., Işıksal Bostan, M., & Tunç Pekkan, Z. (2023). Sixth-grade students' pattern generalization approaches. *Journal of Pedagogical Research, 7*(5), 136–155. https://doi.org/10.33902/JPR.202316928

Kania, N., Suryadi, D., Kusumah, Y. S., Dahlan, J. A., Nurlaelah, E., & Elsayed, E. E. (2024). Comparative praxeology: Assessing high-level cognitive skills in timss and indonesian national examinations. *International Journal of Applied Learning and Research in Algebra, 1*(1), 25–47. https://doi.org/10.56855/algebra.v1i1.1160

Kemendikbudristek. (2024). *Salinan keputusan kepala badan standar, kurikulum, dan asesmen pendidikan kementerian pendidikan, kebudayaan, riset, dan teknologi (032/H/KR/2024)*.

Kementerian Pendidikan Dasar dan Menengah. (2025). *Hasil TKA: Statistik nasional*. https://tka.kemendikdasmen.go.id/hasiltka/

Kieran, C. (2022). The multi-dimensionality of early algebraic thinking: Background, overarching dimensions, and new directions. *ZDM – Mathematics Education, 54*(6), 1131–1150. https://doi.org/10.1007/s11858-022-01435-6

Kivunja, C. (2015). Exploring the pedagogical meaning and implications of the 4cs "super skills" for the 21st century through bruner's 5e lenses of knowledge construction to improve pedagogies of the new learning paradigm. *Creative Education, 6*(2), 224–239. https://doi.org/10.4236/ce.2015.62021

Kularajan, S., Campbell, T., Roth, L., & Bahajj, A. (2026). Modeling-oriented student noticing: An exploratory investigation. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-025-10473-2

Lestari, W., Setianingsih, R., & Sari, Y. M. (2025). Exploring students' statistical problem-solving processes using GeoGebra Classroom: A junior high school case study. *Journal Focus Action of Research Mathematic (Factor M), 8*(1), 98–112. https://doi.org/10.30762/f_m.v8i1.5004

Lobato, J., Rhodehamel, B., & Hohensee, C. (2012). "Noticing" as an alternative transfer of learning process. *Journal of the Learning Sciences, 21*(3), 433–482. https://doi.org/10.1080/10508406.2012.682189

Lobato, J., Hohensee, C., & Rhodehamel, B. (2013). Students' mathematical noticing. *Journal for Research in Mathematics Education, 44*(5), 809–850. https://doi.org/10.5951/jresematheduc.44.5.0809

Machdalena, S., Putri, S., & Putri, R. K. (2022). Profil pemecahan masalah matematika berdasarkan teori polya ditinjau dari kemampuan matematika siswa. *Jurnal Cendekia: Jurnal Pendidikan Matematika, 6*(2), 1776–1787. https://doi.org/10.31004/cendekia.v6i2.1090

Masito, D., Tuhu, S., Morin, S., & Saputri, V. (2024). The relationship between indicators of mathematical problem-solving ability and performance on the PISA Test: Systematic literature review. *Didaktika: Jurnal Kependidikan, 13*(4). https://jurnaldidaktika.org/contents/article/view/1052

Miles, M. B., Huberman, A. M., & Saldana, J. (2014). *Qualitative data analysis: A methods sourcebook* (3rd ed.). SAGE Publications.

Mora, M., Ramírez, R., Gutiérrez, A., & Jaime, A. (2024). Traits of generalization of problem solution methods exhibited by potential mathematically gifted students when solving problems in a selection process. *ZDM – Mathematics Education*. https://doi.org/10.1007/s11858-024-01625-4

Muslim, M., Nusantara, T., Sudirman, S., & Irawati, S. (2024). The causes of changes in student positioning in group discussions using Polya's problem-solving and commognitive approaches. *Eurasia Journal of Mathematics, Science and Technology Education, 20*(9). https://doi.org/10.29333/ejmste/15148

OECD. (2023). *PISA 2022 results (Volume I): The state of learning and equity in education*. OECD Publishing. https://doi.org/10.1787/53f23881-en

Polya, G. (1973). *How to solve it* (2nd ed.). Princeton University Press.

Pradana, L. N. (2024). Problem-solving strategy: Mathematical problem-solving model within the Polya' framework. *KnE Social Sciences*. https://doi.org/10.18502/kss.v9i6.15327

Radford, L. (2010). The eye as a theoretician: Seeing structures in generalizing activities. *For the Learning of Mathematics, 30*.

Rahman, M. M. (2019). 21st century skill "problem solving": Defining the concept. *Asian Journal of Interdisciplinary Research*, 64–74. https://doi.org/10.34256/ajir1917

Risit, A. H., Sudirman, & Sulandra, I. M. (2025). Focuses of students' creativity in constructing quadratic functions through pattern generalization. *Jurnal Elemen, 11*(2), 346–362. https://doi.org/10.29408/jel.v11i2.27894

Riyadi, Syarifah, T. J., & Nikmaturrohmah, P. (2021). Profile of students' problem-solving skills viewed from Polya's four-steps approach and elementary school students. *European Journal of Educational Research, 10*(4), 1625–1638. https://doi.org/10.12973/EU-JER.10.4.1625

Sandelowski, M. (2000). Focus on research methods: Whatever happened to qualitative description? *Research in Nursing and Health, 23*(4), 334–340. https://doi.org/10.1002/1098-240X(200008)23:4<334::AID-NUR9>3.0.CO;2-G

Szabo, Z. K., Körtesi, P., Guncaga, J., Szabo, D., & Neag, R. (2020). Examples of problem-solving strategies in mathematics education supporting the sustainability of 21st-century skills. *Sustainability, 12*(23), 1–28. https://doi.org/10.3390/su122310113

Tanudjaya, C. P., & Doorman, M. (2020). Examining higher order thinking in Indonesian lower secondary mathematics classrooms. *Journal on Mathematics Education, 11*(2), 277–300. https://doi.org/10.22342/jme.11.2.11000.277-300

Toh, T. L., Santos-Trigo, M., Chua, P. H., Abdullah, N. A., & Zhang, D. (2024). Problem posing and problem solving in mathematics education: International research and practice trends. In *Problem posing and problem solving in mathematics education: International research and practice trends*. Springer Nature. https://doi.org/10.1007/978-981-99-7205-0

Utami, N. S., Prabawanto, S., & Suryadi, D. (2023). How students generate patterns in learning algebra? A focus on functional thinking in secondary school students. *European Journal of Educational Research, 12*(2), 913–925. https://doi.org/10.12973/eu-jer.12.2.913

Wijaya, T. T., Hidayat, W., Hermita, N., Alim, J. A., & Talib, C. A. (2024). Exploring contributing factors to pisa 2022 mathematics achievement: Insights from indonesian teachers. *Infinity Journal, 13*(1), 139–156. https://doi.org/10.22460/infinity.v13i1.p139-156

Wilkie, K. J. (2016). Students' use of variables and multiple representations in generalizing functional relationships prior to secondary school. *Educational Studies in Mathematics, 93*(3), 333–361. https://doi.org/10.1007/s10649-016-9703-x

Wilkie, K. J. (2022). Generalization of quadratic figural patterns: Shifts in student noticing. *Journal of Mathematical Behavior, 65*. https://doi.org/10.1016/j.jmathb.2021.100917

Wilkie, K. J. (2024). Coordinating visual and algebraic reasoning with quadratic functions. *Mathematics Education Research Journal, 36*(1), 33–69. https://doi.org/10.1007/s13394-022-00426-w

Wulan, E. R., & Anggraini, R. E. (2019). Gaya kognitif field-dependent dan field-independent sebagai jendela profil pemecahan masalah polya dari siswa smp. *Journal Focus Action of Research Mathematic (Factor M), 1*(2). https://doi.org/10.30762/f m.v1i2.1503

Conflict of Interest Statement: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.