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Analysis of Mathematical Problem Solving Ability of Class XI Students MAN1 Kota Kediri with IDEAL Steps In Term of Learning Independence

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Keywords

Kemampuan pemecahan masalah, Langkah IDEAL, Kemandirian belajar.

Problem Solving Ability, IDEAL Steps, Learning Independence

ABSTRACT

Kemampuan pemecahan sangat dipengaruhi oleh banyak faktor, salah satunya adalah kemandirian belajar. Penelitian ini bertujuan untuk mengetahui kemampuan pemecahan masalah matematis siswa kelas XI MAN 1 Kota Kediri ditinjau dari kemandirian belajar menggunakan tahapan IDEAL. Jenis penelitian ini adalah deskriptif kualitatif dengan subjek penelitiannya adalah siswa kelas XI MIPA 3 MAN 1 Kota Kediri. Teknik pengumpulan data menggunakan tes, angket, observasi, dan wawancara. Instrumen penelitian menggunakan tes pemecahan masalah matematis, rubrik observasi kemandirian belajar, serta angket kemandirian belajar. Tahapan dalam analisis data yaitu reduksi data, penyajian data, dan penarikan kesimpulan. Hasil penelitian menunjukkan bahwa terdapat 3 kategori kemampuan pemecahan masalah matematis siswa jika ditinjau dari kemandirian belajar yaitu rendah, sedang, dan tinggi. Berdasarkan tahapan IDEAL dalam pemecahan masalah didapatkan bahwa siswa berkemandirian belajar rendah memenuhi 2 indikator langkah pemecahan IDEAL. Sedangkan siswa berkemandirian belajar sedang dan tinggi memenuhi seluruh indikator langkah pemecahan *IDEAL*.

Solving ability is strongly influenced by many factors, one of which is learning independence. This study aims to determine the student's mathematical problem solving ability of Class XI MAN 1 Kota Kediri with IDEAL steps riviewed from learning independence. The method used in this study is qualitative descriptive method. The subjects in this study are students of class XI MIPA 3 MAN 1 Kota Kediri. The data collection methods used are test of mathematical problem solving, questionnaries, observation, and interview. Data analysis techniques used are data reduction, data presentation, and verification. The results of this study indicate that there are 3 categories of student's mathematical problem solving ability in terms of learning independence, namely: low, moderate, and high. Students with low learning independence could fulfill 2 indicators of IDEAL problem solving. Students with medium and high learning independence could fulfill all indicators of IDEAL problem solving.

INTRODUCTION

One of the abilities that students need to learning mathematics is problem solving ability. When students are in the process of developing ideas, constructing new knowledge, and at the stage of developing mathematical skills, the first step taken is problem solving itself (NCTM, 2000). Problem solving is a series of activities to overcome various difficulties encountered to achieve a goal (Sumartini, 2016).

According to the results of Progamme for International Students Assessment (PISA) in 2015, Indonesia ranks 62nd out of 70 countries in the category of math ability with an average score of 386. While the results of PISA in 2018, Indonesia ranks 73rd out of 79 countries in the category of math ability with an average score of 379 (OECD, 2019). It can be seen that Indonesia experienced a decline in PISA results from 2015 to 2018. This shows that there is a problem with students' problem solving ability. In addition, facts in the field shows that there are still indicators of problem solving that haven't been fulfilled well by students.

Based on the researcher's initial observation through an interview with a mathematics teacher of class XI MAN 1 Kota Kediri, the teacher explained that most students still have low mathematical problem solving ability. This is evidenced by the existence of a number of students who have not been able to solve math problems optimally. One of them is the problem that leads to limit function material.

The problem solving step model to investigate students' mathematical problem solving ability in this study is IDEAL solution step (Identify problem, Define goal, Explore possible strategies, Anticipate outcomes and act, Look back and learn) which was introduced by John D. Bransford and Barry S. Stein in 1984 as a form of approach to help someone in solving problems (Bransford & Stein, 1984). In addition to using solution step model, students can know the level of solving ability from learning independence. Learning independence is a learning activity that carried out by a person using independence and without dependence on others as an improvement in the field of knowledge, skills, and even development in one's achievements (Hidayat, Nadine, Ramadhan, & Rohaya, 2020).

Based on research by Maimuanah, Roza, and Sulistyani (2020) about the relationship between learning independence and mathematical problem solving ability, a positive relationship was found between of them. The results of his research calculations show that learning independence and students' mathematical problem solving ability have a positive relationship of 0.764 with a significance value of $0.000 < 0.05$. This shows that learning independence can be used as one of the causal factors that can affect mathematical problem solving ability.

This is supported by Mayasari and Rosyana's research (2019) which found that there is a linear relationship between learning independence and students' mathematical problem solving ability. The research is also supported by research Ambiyar, Aziz, and Delyana (2020) related to relationship between student's learning independence and mathematical problem solving ability, where the results of his research show that there is a positive relationship between learning independence and mathematical problem solving ability from the results of simple linear regression and pearson correlation tests. If students have high learning independence, so students' problem solving ability are good and this applies vice versa. There is no research related to mathematical problem solving ability and student's learning independence with IDEAL steps from the literature survey, motivated the researcher to conduct a study entitled " Analysis of Mathematical Problem Solving Ability of Class XI Students MAN 1 Kota Kediri with IDEAL Steps In Term of Learning Independence."

METHODS

This type of research is descriptive qualitative research. The research subjects are students of class XI MIPA 3 MAN 1 Kota Kediri. The research sampling process began with categorization of student's learning independence (low, medium, and high) through observation and learning independence questionnaire. Then, each sample of learning independence is asked to take test and will be interviewed for problem solving on the test. Based on the research, researchers were only able to find 2 students in each category of learning independence. The data collection techniques used include tests, observations, questionnaires, and interviews.

The triangulation used is triangulation of data collection techniques (test results with the results of student problem solving interviews). The research data analysis techniques include data reduction, data presentation, and conclusion drawing. The main research instrument is the researcher himself and his supporting instruments include problem solving test instruments, interview sheet, observation sheet, and questionnaire of learning independence. The research instrument have been tested for content validity by several expert lecturers of IAIN Kediri and a mathematics teacher at MAN 1 Kota Kediri. Calculation of the results of content validity using Aiken's formula $\left(\frac{\sum s}{n(c-1)}\right)$. After validity calculation, researcher determined validity of the instrument items with the Aiken's index criteria and it was found that all items of the instrument were suitable for research because it fulfills high to very high criteria.

The average value of the validator's Aiken's index criteria for questionnaires, interviews, observation and tests were 0,748 (high), 0,889 (very high), 0,815 (very high), 0,889 (very high). The questionnaire and observation grids for learning independence have sub-indicators of motivation in learning, self-confidence, discipline in learning, responsibility in learning, and active in learning. The form of student mathematical problem solving test question related to function limits are in Table 1.

Table 1. Mathematical Problem Solving Test Numbers and Questions

No.	Mathematical Problem Solving Test Question
1	There exists $\lim_{x\to 6} f(x) = 8$, $\lim_{x\to 6} g(x) = 4a$, and $h(x) = 2$. If $\lim_{x\to 6}\left(\left(f(x)^{h(x)} + g(x) - 4\right) = 61\right.$, what value of a satisfies the limit equation of the function?
2	There are two limits, namely $\lim_{x\to 2} \frac{1+x-\sqrt{x^2+x+3}}{2-x}$ and $\lim_{x\to -3} \frac{4x^2+12x}{5x+15}$. What is the sum of the two values of the of the limit?
3	Jack rides his motorcycle to school. He drives the motorcycle at a certain speed, so that the distance travelled at any time is given by $s(t) = 2t^2 - 22t$, s in metre and t in second. What is the instantaneous speed of the Jack’s motorcycle when t approaches 15 seconds?
4	Saka heats a metal plate. It’s observed that the heated metal plate expands in area as a function of time $f(t) = 0{,}36t^2 + bt\ (cm^2)$. If rate of change of the area as t approaches 10 minutes can

No.	Mathematical Problem Solving Test Question
	be formulated by $v(t) = \lim_{t\to 10} \frac{0{,}36t^2+bt-40}{t-10} = 7{,}6\ cm^2/minute$, determine the value of b and form equation for rate of change of the Saka's metal area when t is close to 3 minutes!

RESULTS AND DISCUSSION

The following is a list of research subject that were successfully obtained in the study.

Table 2. List of Research Subject

No.	Research Subject Code	Learning independence Category
1	A32	Low
2	A33	Low
3	A9	Medium
4	A22	Medium
5	A24	High
6	A25	High

In the next stage, the researcher will analyze some of the description test result (numbers 2 and 4), six research subjects who can represent mathematical problem solving ability of each learning independence category.

Mathematical Problem Solving Ability of Students with Low Learning Independence

The problem solving test results of student A32 and student A33 are shown in Figure 2 to Figure 4.

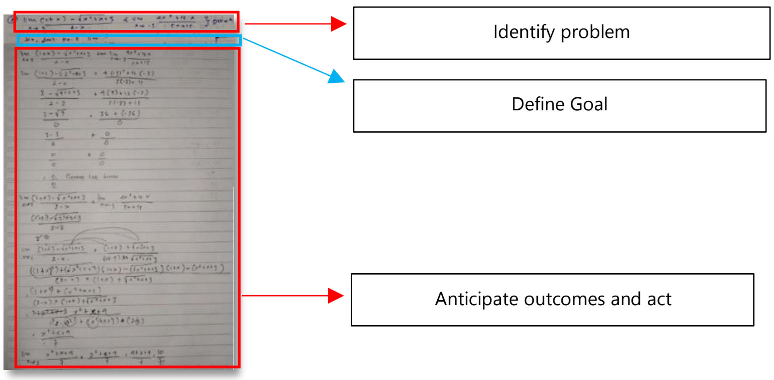

Figure 1. Results of Student A32's Test on Problem Number 2

Figure 1 shows the results of student A32's work on problem number 2, it can be seen that student was able to identify problem and define goal the problem. The following is a transcript of the student's interview on problem solving number 2.

P	:	*Describe the information you know in the problem?*
A32	:	*It's known the limit function x that is close to 2 namely* $\lim\limits_{x\to 2}\frac{1+x-\sqrt{x^2+x+3}}{2-x}$ *and limit function x close to* -3 *namely* $\lim\limits_{x\to -3}\frac{4x^2+12x}{5x+15}$
P	:	*What do we have to find in this problem?*
A32	:	*The sum of the two limits.*
P	:	*How to solve the problem?*
A32	:	*First, find the value of lim... x* $= 2$ *input to* $\lim\limits_{x\to 2}\frac{1+x-\sqrt{x^2+x+3}}{2-x}$ *and input* $x = -3$ *to* $\lim\limits_{x\to -3}\frac{4x^2+12x}{5x+15}$*, and then add results of the values. The result is* $\frac{0}{0}$*. Thens* $\lim\limits_{x\to 2}\frac{1+x-\sqrt{x^2+x+3}}{2-x}$ *This is multiplied by irrational common root and the other lim use factoring.*
P	:	*Did you write this strategy down?*
A32	:	*No*
P	:	*How did you solve the problem as a whole?*
A32	:	*I did the addition,* $\lim\limits_{x\to 2}\frac{1+x-\sqrt{x^2+x+3}}{2-x}$ *plus* $\lim\limits_{x\to -3}\frac{4x^2+12x}{5x+15}$ *becomes an indeterminate form. Then,* $\lim\limits_{x\to 2}\frac{(1+x)-\sqrt{x^2+x+3}}{2-x}$ *multiplied by irrational root friend. After calculation, it's equal to* $\frac{10}{7}$*. Then we use the factoring method. So, for* $\lim\limits_{x\to -3}\frac{4x^2+12x}{5x+15}$ *separate .... So that leaves* $\frac{4x}{5}$*, then put it into* $\lim\limits_{x\to -3}\frac{4(-3)}{5} = -\frac{12}{5}$*. Then the value of 2 limits is entered, the result is* $-\frac{34}{35}$
P	:	*Did you carry out the steps of the solution according to the strategy you defined?*
A32	:	*I did*
P	:	*Did you check the solution and your answer?*
A32	:	*Not yet.*
P	:	*What is the answer you get?*
A32	:	*The sum of the two limits is* $-\frac{34}{35}$*.*

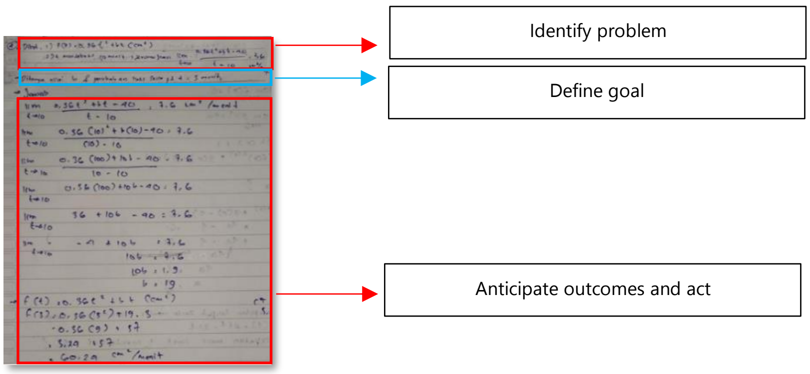

Figure 2. Results of Student A32's Test on Problem Number 4

Figure 2 shows the results of student A32's work on problem number 4, it can be seen that student didn't fulfill all IDEAL solution steps. The following is a transcript of the student interview on problem solving number 4.

P	:	*Describe the information you know in the problem?*
A32	:	*Increase in area as a function of time* $f(t) = 0{,}36t^2 + bt\ (cm^2)$*. Then we know that t is close to 10 minutes and can be formulated as can be formulated with* $v(t) = \frac{0{,}36t^2+bt-40}{t-10} = 7{,}6cm^2/minute.$
P	:	*What do we need to find in the problem?*
A32	:	*The value of b and formula for equation of the rate of change of Saka's metal surface at t* $=$ 3 *minutes.*
P	:	*How do you solve the problem?*
A32	:	*By using limit equation and function* $f(t)$*.*
P	:	*How do you solve the problem as a whole?*
A32	:	*So, 0,36 ...* t^2 *is replaced by* 10^2 *plus* bt *filled with 10 min 40 per 10 min 10 equal to 7,6* $cm^2/minute$*. Then* $\frac{0{,}36(100)+10b-40}{10-10} = 7{,}6$*. Then* $36 + 10b - 40 = 7{,}6$*. . I looked for a result close to 7.6 from the equation and the result is 19. Then I found b in* $f(t)$*. So for find t of the Saka metal at t 3 minutes.* $f(t)$ *equals to* $0{,}36t^2 + bt\ (cm^2)$*. t t is replaced by 3 ... b is 19 ...* $so, 0{,}36(3)^2 + 19(3) = 60{,}24\ cm^2/minute.$
P	:	*Did you solve the problem with the strategy you decided?*
A32	:	*I did*
P	:	*Did you check the solution and your answer?*
A32	:	*Not yet, not enough time.*

P	:	*What is the answer you get?*
A32	:	*The value of b = 19, the formula $f(t) = 60{,}24\ cm^2/minute$*

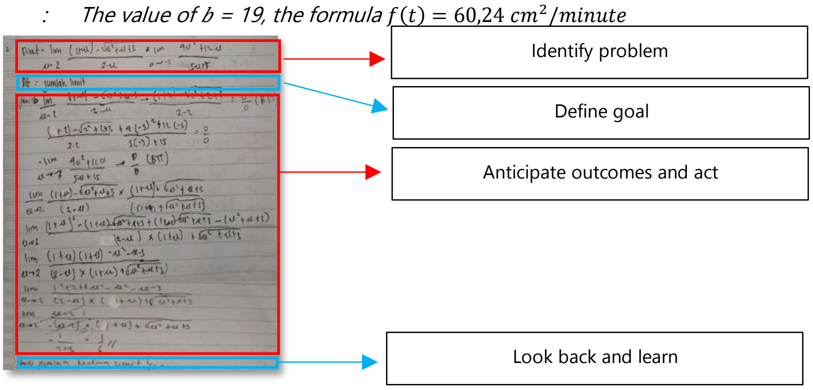

Figure 3. Results of Student A33's Test on Problem Number 2

Figure 3 shows the results of student A33's work on problem number 2, it can be seen that student was able to identify problem and define goal the problem. The following is a transcript of the student interview on problem solving number 2.

P	:	*Describe the information you know in the problem?*
A33	:	*First,* $\lim\limits_{x\to 2}\frac{(1+x)-\sqrt{x^2+x+3}}{2-x}$ *and* $\lim\limits_{x\to -3}\frac{4x^2+12x}{5x+15}$ *, that's what is known.*
P	:	*What do we have to find in the problem?*
A33	:	*The sum of the values from two limits.*
P	:	*How do you solve it?*
A33	:	*Use times irrational root friend, the problem is the initial result of the number is* $\frac{0}{0}$.
P	:	*What is the beginning?*
A33	:	*That* $x = 2$ *input to first limit and add* $x = -3$ *to second limit, the result is* $\frac{0}{0}$.
P	:	*Did you write this strategy down?*
A33	:	*No, Miss ... answer directly*
P	:	*How did you solve the problem as a whole?*
A33	:	*First, I entered two limits.* $\lim\limits_{x\to 2}\frac{(1+x)-\sqrt{x^2+x+3}}{2-x} = \frac{0}{0}$ *Then* $\frac{(1+2)-\sqrt{2^2+2+3}}{2-2} + \frac{4(-3)^2+12(-3)}{5(-3)+15}$ ... *the result is* $\frac{0}{0}$ *Then,* $\lim\limits_{x\to -3}\frac{4x^2+12x}{5x+15} = \frac{0}{0} =$ *indeterminate form ... not found. Then* $\lim\limits_{x\to 2}\frac{(1+x)-\sqrt{x^2+x+3}}{2-x}$ *times* $\frac{(1+x)+\sqrt{x^2+x+3}}{(1+x)+\sqrt{x^2+x+3}}$ *irrational root friend. The result* $\lim\limits_{x\to 2}$ *is* $\frac{1^2+2+x-x^2-x-3}{(2-x)(1+x+\sqrt{x^2+x+3})} = -\frac{1}{3+3} = \frac{1}{6}$
P	:	*For the part* $\lim\limits_{x\to -3}\frac{4x^2+12x}{5x+15}$ *Which part is this?*
A33	:	*Not yet, miss.*
P	:	*Have you carried out the steps of the solution according to the strategy you decided?*
A33	:	*Already*
P	:	*Have you rechecked the solution and your answer again?*
A33	:	*Not yet, just to be sure.*
P	:	*What is the answer you get?*
A33	:	*The other limit is* $\frac{1}{6}$

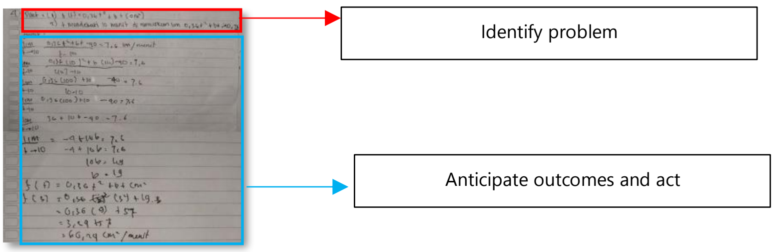

Figure 4. Results of Student A33's Test on Problem Number 4

Figure 4 shows the results of student A33's work on problem number 4, it can be seen that student didn't fulfill all IDEAL solution steps. The following is a transcript of the student's interview on problem solving number 4.

P	:	*Describe the information you know in the problem?*
A33	:	*We know that,* $f(t) = 0{,}36t^2 + bt(cm^2)$ *is the increase in area as a function of time. There is more,* t *is close to 10 minutes formulated by* $lim\, 0{,}36t^2 + bt - 40 = 7t$*. Uh, 7,6.*
P	:	*What do you have to find in the problem?*
A33	:	*What is asked is ... find the value of* b *and write formula for rate of change of Saka's metal surface at* t *3 minutes.*
P	:	*Did you write it down?*
A33	:	*No, I didn't. I was in a hurry because the time was up. I forgot to write it down.*
P	:	*What is your method/strategy to solve the problem?*
A33	:	*The strategy, just try.*
P	:	*What is your process in solving the problem as a whole?*
A33	:	*The answer* $\lim_{t\to 10} \frac{0{,}36t^2+bt-40}{t-10} = 7{,}6\ lim/minute$*. Now,* $\lim_{t\to 10} \frac{0{,}36(10)^2+b(10)-40}{10-10} = 7{,}6$ *.Then* $\lim_{t\to 10} 36 + 10b - 40 = 7{,}6$ *. Then* $-4 + 10b = 7{,}6$*. So* $b = 19$*. Using the formula from the previous question* $f(t) = 0{,}36t^2 + bt\ cm^2$*. So* $f(3) = 0{,}36\ (3)^2 + 19.3 = 60{,}24\ \ cm^2/\ minute$*. So the answer is the area*
P	:	*Did you solve the problem according to your strategy?*
A33	:	*I did.*
P	:	*Did you double check your solution process and your answer?*
A33	:	*I was sure of the answer. There was minimal correction because the time was short.*
P	:	*What is the answer you get?*
A33	:	$b = 19\ and\ the\ area\ is\ 60{,}24\ \ cm^2/\ minute.$

Based on the results of the analysis of test data and interviews with students A32 and A33, students with low learning independence in problem solving only fulfill 2 indicators of the IDEAL steps (identify problem and definy goal). The students fulfill indicators identify problem and define goal because students are able to find the known and questionable information in the problem correctly. Although, sometimes students are also incomplete and incorrect in writing known information as well as writing required information especially in story problems. This is in line with research of Bunga Suci Bintari Rindyana cited in (Fitriyani & Yusnia, 2017) that the errors made by some students in solving story problems, namely not writing known information and asked in the problem, and also writing known information and asked isn't in accordance with the request of the question. This is also in line with the research of Nurussafa'at, Riyadi, and Sujadi (2016) that student's errors in solving story problems is incomplete in writing what is asked and not writing what is known in the problem

On the indicator of explore possible strategies, students are sometimes able to plan problem solving strategies and not infrequently students are unable to develop a solution strategy. The reason students weren't able to develop a problem solving strategy is because they didn't know formula for solving the problem and because they weren't able to develop the solution steps for story problems. For indicator of anticipate outcomes and act, students are sometimes able to write process of solving the problem until they get solution correctly. However, it isn't uncommon for students unable to write the solution process completely and accurately. The reason of it is students are only able to write several part of the solution process about sum of limits, students use wrong solution strategy for limit story problem, and students make errors calculating in the solution so it make the answer is less accurate. This is in line with Farida's research (2015), which states that student errors in solving story problems are students making errors in planning what to do because they can't determine formula for solving problem, and students' miscalculations in finding the answer.

In the look back and learn indicator, students tend to haven't checked the solution of the problem because students feel that they already have the answer of the problem. This is in line with the research of Hasanah and Imami (2022) that students with low learning independence don't recheck after getting the answer of the problem.

Mathematical Problem Solving Ability of Students with Medium Learning Independence

The results of the problem solving test of student A9 and student A22 are shown in Figure 5 to Figure 8.

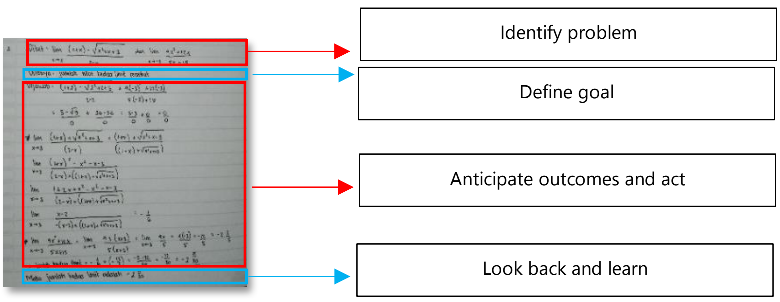

Figure 5. Results of Student A9's Test on Problem Number 2

Figure 5 shows the results of student A9's work on problem number 2, it can be seen that student wasn't write possible strategy for the problem. The following is a transcript of the student's interview about problem solving number 2.

P	:	Can you mention the information you know in the problem?
A9	:	*We know that* $\lim_{x\to 2}\frac{1+x-\sqrt{x^2+x+3}}{2-x}$ *and* $\lim_{x\to -3}\frac{4x^2+12x}{5x+15}$.
P	:	*What do you have to find in this problem?*
A9	:	*The sum of the two limits.*
P	:	*How do you solve the problem?*

A9 : *First, I entered the value of x from limit, but the result was $\frac{0}{0}$. Then, since the result $\frac{0}{0}$, I use the method of irrational root friend and factoring.*
P : *How do you solve the problem as a whole?*
A9 : *First thing is to enter directly from lim x is close to 2 and -3, then it's calculated to be equal to $\frac{0}{0}$ indeterminate form. Then use the multiplication method of irrational root friend, from $\lim\limits_{x\to 2}\frac{1+x-\sqrt{x^2+x+3}}{2-x}$, so $-\frac{1}{6}$. The second method uses factoring, it becomes $-2\frac{2}{5}$. Then add two limits, $-\frac{1}{6}+\left(-\frac{12}{5}\right)$ is $-2\frac{17}{30}$.*
P : *Did you solve the problem according to the strategy you defined?*
A9 : *I did.*
P : *Did you check your solution and your answer?*
A9 : *Not yet*
P : *What is the answer you get?*
A9 : $-2\frac{17}{30}$

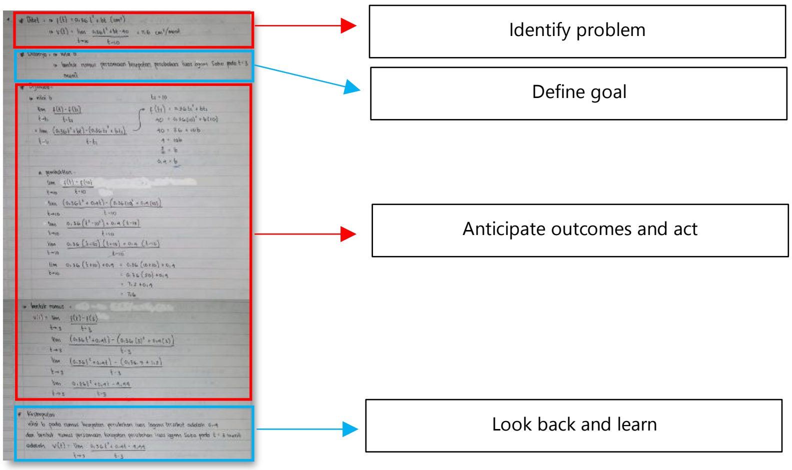

Figure 6. Results of Student A9's Test on Problem Number 4

Figure 6 shows the results of student A9's work on problem number 4, it can be seen that student wasn't write possible strategy for the problem. The following is a transcript of the student's interview about problem solving number 4.

P : *What information do you know about the problem?*
A9 : *Time function $f(t) = 0{,}36t^2 + bt(cm^2)$. Then we know formula for the change in velocity of the metal surface is $v(t) = \lim\limits_{t\to 10}\frac{0{,}36t^2+bt-40}{t-10} = 7{,}6\ cm^2/minute$*
P : *What do you need to find in the problem?*
A9 : *The value of b and formula for the change in area of Saka's metal at t = 3 minutes.*
P : *How do you solve the problem?*
A9 : *First, find the value of b. The rate of change $= \lim\limits_{t\to t_1}\frac{f(t)-f(t_1)}{t-t_1}$. Where t_1 is initial time. Second, to find form of the formula for the change in the rate of change in the area of the saka metal when $t = 3$ minutes (pointing to $\lim\limits_{t\to t_1}\frac{f(t)-f(t_1)}{t-t_1}$) use that formula. Only for t_1 is replaced by 3.*
P : *What is your process for solving the problem as a whole?*
A9 : *First ... find the value of b using the formula rate of change $=\lim\limits_{t\to t_1}\frac{f(t)-f(t_1)}{t-t_1} = \lim\limits_{t\to t_1}\frac{(0{,}36t^2+bt)-(0{,}36t_1^2+bt_1)}{t-t_1}$. For the value of $f(t_1)$is 40 and we know $t_1 = 10$. So, we put in $0{,}36t_1^2 + bt_1$ ... The result is $36 + 10b = 40, b = \frac{4}{10}$, converted to decimals to 0.4. To find the formula for the rate of change of Saka's metal area at t = 3 minutes use the same formula. But, for t_1 change with 3. $\lim\limits_{t\to 3}\frac{f(t)-f(3)}{t-3} = \lim\limits_{t\to 3}\frac{(0{,}36t^2+0{,}4t)-(0{,}36(3)^2+0{,}4(3))}{t-3}$and you get $\lim\limits_{t\to 3}\frac{0{,}36t^2+0{,}4t-4{,}44}{t-3}$*
P : *Did you carry out the steps of solving the problem according to the strategy you decided?*
A : *I did*
P : *Did you check your solution process and your answer?*
A9 : *I did. To check the value of b, I'm check through the calculation and proof, and the others I tried to calculate again, maybe there is a miscalculation or not.*
P : *What is the answer you get?*

A9 : *The value of b is 0.4, then the formula for the rate of change of Saka's metal area when t = 3 minutes is* $v(t) = \lim_{t\to 3} \frac{0{,}36t^2+0{,}4t-4{,}44}{t-3}$

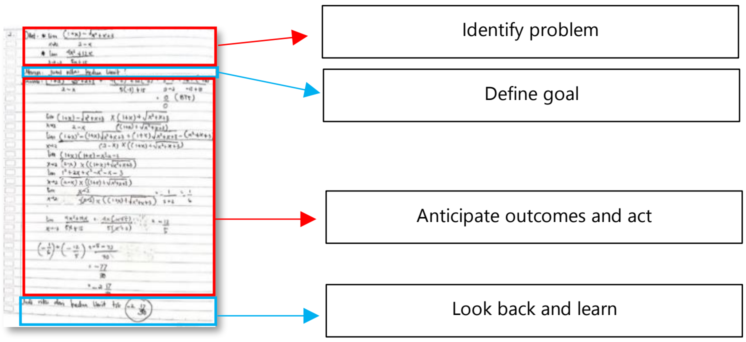

Figure 7. Results of Student A22's Test on Problem Number 2

Figure 7 shows the results of Student A22's work on problem number 2, it can be seen that student wasn't write possible strategy for the problem. The following is a transcript of the student's interview about problem solving number 2.

P : *Describe the information you know in the problem?*
A22 : *In the problem, 2 limits are known, the first is* $\lim_{x\to 2} \frac{(1+x)-\sqrt{x^2+x+3}}{2-x}$ *and the second limit is* $\lim_{x\to -3} \frac{4x^2+12x}{5x+15}$.
P : *What do you need to find out from this question?*
A22 : *The sum of the two limits.*
P : *How do you solve the problem?*
A22 : *First, we find the limit value in the form of a determinete/indeterminete form of the two limits by finding the value of x in both limits, so we get that both limits have an indeterminate form. Second, we find the result of each limit. The result of* $\lim_{x\to 2} \frac{(1+x)-\sqrt{x^2+x+3}}{2-x}$, *it is* $-\frac{1}{6}$ *because limit is multiplied by irrational root friend. Then the second limit is* $\lim_{x\to -3} \frac{4x^2+12x}{5x+15}$, *we factor it and get* $-\frac{12}{5}$. *After obtaining the value of each limit, we add them together, namely* $-\frac{1}{6}+\left(-\frac{12}{5}\right) = -\frac{77}{30}$ *or* $-2\frac{17}{30}$
P : *For the strategy you made before, did you write it down on paper?*
A22 : *No ... the question is done immediately*
P : *What is your process for solving the problem as a whole?*
A22 : *Well, first we find the limit in the form of a determinete/indeterminete form, we use substitution method for limit... Then, the results of the two limits substituted are summed ... the result is* $\frac{0}{0}$ *or indeterminate form. Then, econd way, we find the value of the two limits. The first limit* $\lim_{x\to 2} \frac{(1+x)-\sqrt{x^2+x+3}}{2-x}$ *we multiply with irrational root friend, namely* $\frac{(1+x)+\sqrt{x^2+x+3}}{(1+x)+\sqrt{x^2+x+3}}$ *Then we get* $-\frac{1}{6}$, *in the second limit,* $\lim_{x\to -3} \frac{4x^2+12x}{5x+15}$, *we use factoring. Later we get* $-\frac{12}{5}$. *Then the third way ... the value of the two limits we add them. So it is equal to* $-\frac{77}{30}$ *or* $-2\frac{17}{30}$.
P : *Did you go through the steps of solving the problem according to the strategy that you defined?*
A22 : *I did*
P : *Did you check the solution process and your answer?*
A22 : *Yes, I did.*
P : *Explain the checking process that you did!*
A22 : *By checking again from the beginning of the solution process.*
P : *What is the answer you get?*
A22 : *The answer that I get is* $-2\frac{17}{30}$ *or* $-\frac{77}{30}$.

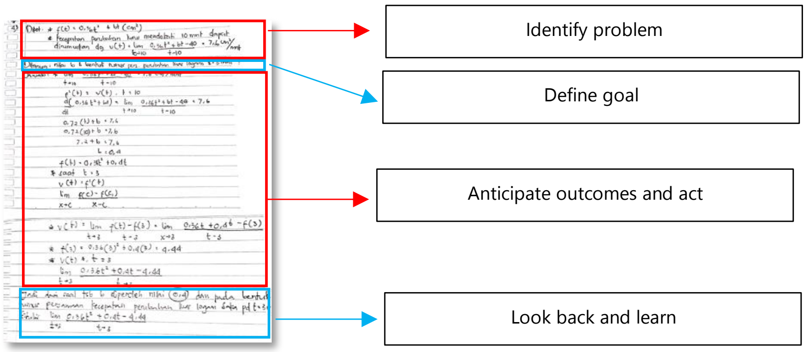

Figure 8. Results of Student A22's Test on Problem Number 4

Figure 8 shows the results of Student A22's work on problem number 4, it can be seen that student wasn't write possible strategy for the problem. The following is a transcript of the student's interview about problem solving number 4.

P	:	*Describe the information that you know in the problem?*
A22	:	*First is* $f(t) = 0{,}36t^2 + bt(cm^2)$. *Second, the rate of change in the area approaching 10 minutes is formulated by* $v(t) = \lim_{t\to 10} \frac{0{,}36t^2+bt-40}{t-10} = 7{,}6\ cm^2/minute$.
P	:	*What do you need to find in the problem?*
A22	:	*The value of b and formula for the equation of the rate of change of Saka's metal area at t = 3 minutes.*
P	:	*How do you solve the problem?*
A22	:	*First, find the value of b. Then substitute the value of b in the formula for the rate of change of area by which is close to 10 minutes. Then continue to determine formula for the equation of the change in metal area when t = 3 minutes*
P	:	*From the method/strategy that you have determined, what is the process of doing it as a whole?*
A22	:	*First, determine the value of b as t approaches 10 minutes is* $\lim_{t\to 10} \frac{0{,}36t^2+bt-40}{t-10} = 7{,}6\ cm^2/minute. f'(t) = n\,v(t)$. *Then* $\frac{d}{dt}(0{,}36t^2 + bt) = \lim_{t\to 10} \frac{0{,}36t^2+bt-40}{t-10} = 7{,}6$. *We get* $0{,}72\,t + b = 7{,}6$. *We substitute* $t = 10$. *The result is* $b = 0{,}4$. *So, the equation is* $f(t) = 0{,}36t^2 + 0{,}4t$. *Second, if* $t = 3$ *so* $v(t) = \lim_{t\to 3} \frac{f(t)-f(3)}{t-3} = \lim_{t\to 3} \frac{0{,}36t^2+bt-f(3)}{t-3}$. *Then, we substitute into* $f(t)$ *formula when* $t = 3$ *minutes. So* $f(t)$ *when* $t = 3$ *minutes is 4,44. Thus, the formula for the change of metal area when t = 3 minutes is* $v(t) = \lim_{t\to 3} \frac{0{,}36t^2+0{,}4t-4{,}44}{t-3}$. *I used this method by trial and error. Using the concept of limit, but because the notation lim x approaches c,* $\lim_{x\to c_1} \frac{f(c)-f(c_1)}{x-c_1}$, *may be* $\lim_{x\to t_1} \frac{f(t)-f(t_1)}{x-t_1}$ *That's derivative notation. There is a connection.*
P	:	*Have you done the steps of solving the problem according to the strategy you defined?*
A22	:	*Yes, I have.*
P	:	*Did you check the solution and your answer?*
A22	:	*Yes, I did. I checked with see the values that I substituted were correct. Then the process of moving the segments. And the calculation.*
P	:	*What is the answer you get?*
A22	:	*b is 0.4 and the equation for the rate of change of metal area in Saka when* $t = 3$ *minutes is* $\lim_{t\to 3} \frac{0{,}36t^2+0{,}4t-4{,}44}{t-3}$.

Based on the results of the analysis of test data and interviews with students A9 and A22, students with medium learning independence in solving problems fulfill all indicators of the IDEAL step. Students fulfill indicators of identify problem and define goal because students are able to write down known information and able to understand required information correctly. This is in line with the research of Ekadiarsi and Khusna (2022) that students with medium learning independence in solving problems are able to write down known information and able to understand the information asked in the problem correctly.

On the indicator of explore possible strategies, there are students who are able to plan the problem solving strategy correctly. However, there are also students who haven't been able to develop the right problem solving strategy because students haven't been able to create a solution plan and don't know formula for the problem. In the anticipate outcomes and act indicator, students are sometimes able to write the solution process until they find solution of the problem correctly. However, there are also students who weren't able to write solution process correctly because students incorrectly construct a solution plan that causes the calculation process to be inaccurate. This is consistent with the research of Pujiastuti and Syahda (2020), where students' errors in creating plans when working on math problems caused errors in the calculation process.

In the look back and learn indicator, there are students who have checked the solution of the problem. There are also students who didn't check the solution, so the answer they got isn't correct. This is in line with the research of Hasanah and Imami (2022) which explains that students with medium learning independence are less thorough in solving problems and don't carry out the process of checking answers, so the answers obtained are less accurate.

Mathematical Problem Solving Ability of Students with High Learning Independence

The results of the problem solving tests of student A24 and student A25 are shown in Figures 9 to figure 12.

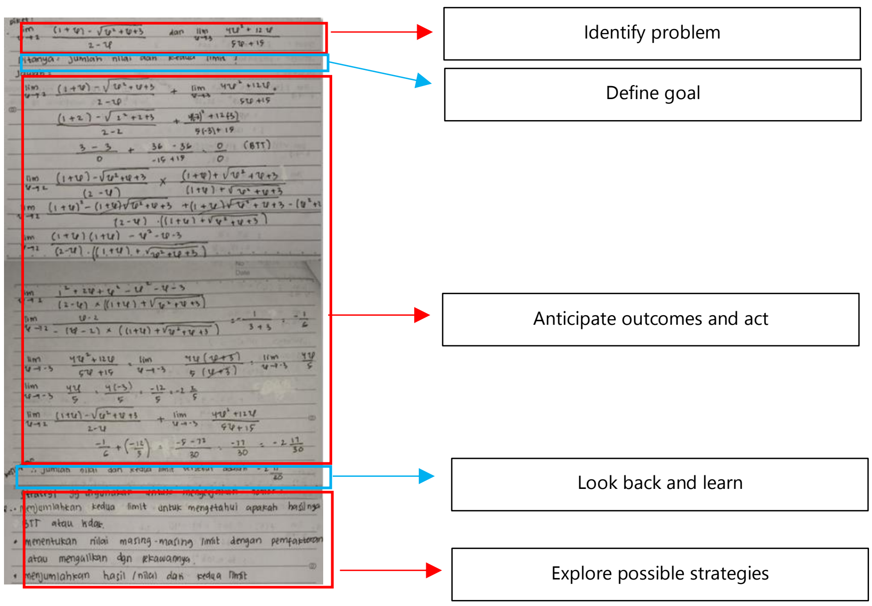

Figure 9. Results of Student A24's Test on Problem Number 2

Figure 9 shows the results of student A24's work on problem number 2, it can be seen that student A24 fulfill all IDEAL solution steps. The following is the transcript of student A24's work on the problem solving number 2.

P : *Describe the information that you know in the problem?*

A24 : *There are 2 limit, the first limit is* $\lim_{x\to 2}\frac{(1+x)-\sqrt{x^2+x+3}}{2-x}$ *and second limit is* $\lim_{x\to -3}\frac{4x^2+12x}{5x+15}$.

P : *What do you need to find in the problem?*

A24 : *The sum of the values from two limits.*

P : *How did you solve the problem?*

A24 : *First, add them directly...* $\lim_{x\to 2}\frac{(1+x)-\sqrt{x^2+x+3}}{2-x}$ *, so x replaced by 2. Then,* $\lim_{x\to -3}\frac{4x^2+12x}{5x+15}$*, so x replaced by* -3*. It turns out that the result is an indeterminate form. Then, find the value of the limit one by one using multiplication with irrational root friend and factoring. Then the values of the two limits are summed.*

P : *Did you write down your strategy or method?*

A24 : *Yes, I wrote it down.*

P : *What is your procedure for solving the problem as a whole?*

A24 : *First, the first limit equation* $\lim_{x\to 2}\frac{(1+x)-\sqrt{x^2+x+3}}{2-x}$ *multiplied by irrational root friend, namely* $\frac{(1+x)+\sqrt{x^2+x+3}}{(1+x)+\sqrt{x^2+x+3}}$ *then we get the result is* $-\frac{1}{6}$*. Then the second limit equation is* $\lim_{x\to -3}\frac{4x^2+12x}{5x+15}$. $\lim_{x\to -3}\frac{4x^2+12x}{5x+15}$*, factored be* $\lim_{x\to -3}\frac{4x(x+3)}{5(x+3)}$*. So,* $(x+3)$ *is eliminated, leaving* $\lim_{x\to -3}\frac{4x}{5}$*. Then x replaced by* -3*. The result is* $-\frac{12}{5}$*. Then just add two limits* $-\frac{1}{6}+\left(-\frac{12}{5}\right)=-2\frac{17}{30}$.

P : *Did you carry out the steps of the solution according to the strategy you defined?*

A24 : *Yes, I did*

P : *Did you check your solution and your answer?*

A24 : *Yes, I did*

P : *What was your vetting process like? Can you explain it?*

A24 : *: I corrected the calculation methods. Min plus is correct or not.*

P : *What is the answer you get?*

A24 : *The result is* $-2\frac{17}{30}$.

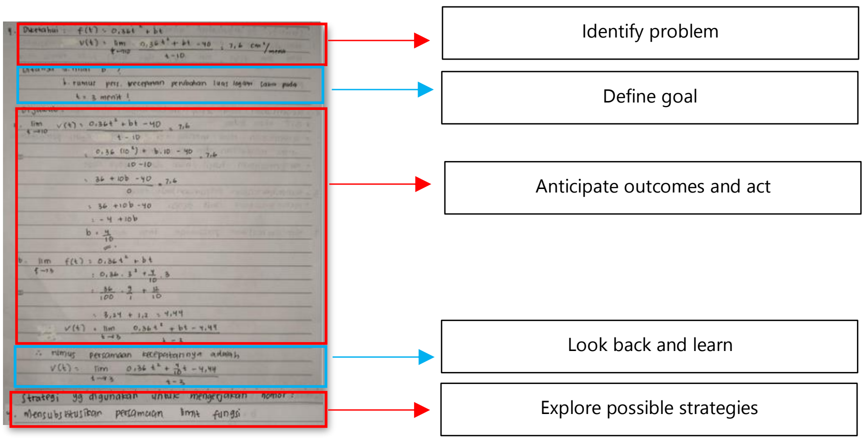

Figure 10. Results of Student A24's Test on Problem Number 4

Figure 10 shows the results of student A24's work on problem number 4, it can be seen that student A24 fulfills all the IDEAL solution steps. The following is the transcript of student A24's work on the problem solving number 4.

P : *Describe the information that you know in the problem?*

A24 : *That known is* $f(t)=0{,}36t^2+bt$ *and* $v(t)=\lim_{t\to 10}\frac{0{,}36t^2+bt-40}{t-10}=7{,}6\ cm^2/minute$.

P : *What do you need to find in the problem?*

A24 : *The value of b and equation for the rate of change of Saka's metal area at t = 3 minutes.*
P : *How do you solve the problem?*
A24 : *Substitute the limit function equation. First, find the value of b, use equation formula (pointing to* $v(t) = \lim_{t\to 10} \frac{0{,}36t^2+bt-40}{t-10} = 7{,}6\ cm^2/minute$*) and b ... equation formula the rate of change of Saka's metal area when* $t = 3$ *minutes using the equation* $f(t) = 0{,}36t^2 + bt$ *and the limit equation* $v(t)$.
P : *Did you write down your strategy?*
A24 : *Yes, just briefly.*
P : *What is your process in solving the problem as a whole?*
A24 : *So,* $\lim_{t\to 10} v(t) = \frac{0{,}36t^2+bt-40}{t-10} = 7{,}6$*. Then, t replaced by 10. The result is* $\frac{36+10b-40}{0} = 7{,}6$*. So the result just* $36 + 10b - 40 = -4 + 10b$*. So,* $b = \frac{4}{10}$*. Then b, the formula for the equation of the rate of change in the area of metal Saka at* $t = 3$ *minutes use the formula* $\lim v(t)$*. So,* $\lim_{t\to 3} f(t) = 0{,}36t^2 + bt$ *... equal to* $0{,}36$ *multiplied by* 3^2 *plus* $\frac{4}{10}$ *multiplied by 3 ... equals 4,44. Then find the velocity,* $v(t) = \lim_{t\to 3} \frac{0{,}36t^2+\frac{4}{10}t-4{,}44}{t-3}$*. It's from formula* $v(t) = \lim_{t\to t_1} \frac{f(t)-f(t_1)}{t-t_1}$
P : *Did you carry out the steps of solving the problem according to the strategy you defined?*
A24 : *I did*
P : *Did you check the solution and your answer?*
A24 : *Yes, I did. I checked the calculation, the writing and the formula.*
P : *What is the answer you get?*
A24 : $b = \frac{4}{10}$ *and the formula* $v(t) = \lim_{t\to 3} \frac{0{,}36t^2+\frac{4}{10}t-4{,}44}{t-3}$

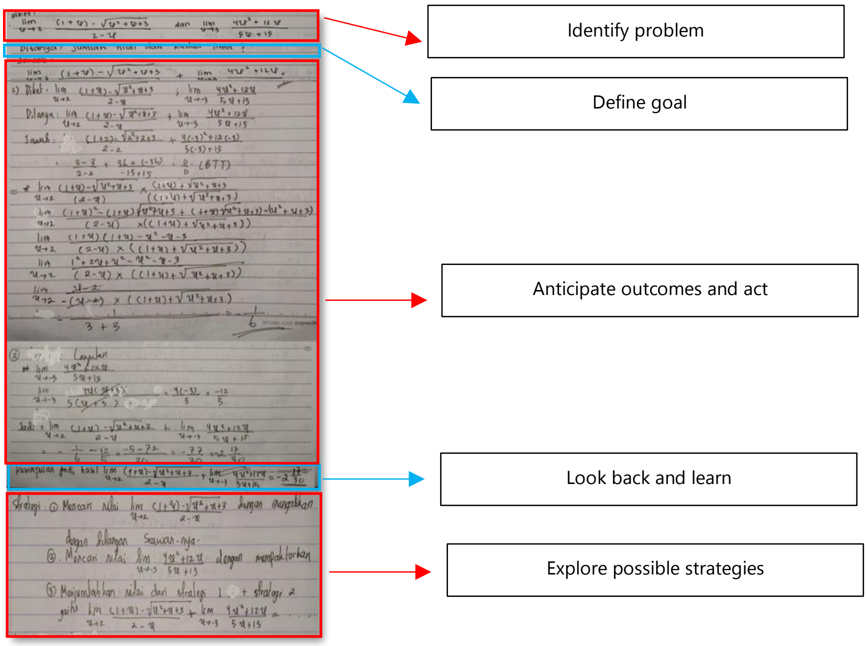

Figure 11. Results of Student A25's Test on Problem Number 2

Figure 11 shows the results of student A25's work on problem number 2, it can be seen that student A25 fulfill all IDEAL solution steps. The following is the transcript of student A25's work on the problem solving number 2.

P : *Describe the information that you know in the problem?*
A25 : *The information is* $\lim_{x\to 2} \frac{1+x-\sqrt{x^2+x+3}}{2-x}$*, then there exists* $\lim_{x\to -3} \frac{4x^2+12x}{5x+15}$
P : *What do you need to find in the problem?*
A25 : *The sum of the values of the two limits.* $\lim_{x\to 2} \frac{1+x-\sqrt{x^2+x+3}}{2-x}$ *is summed with* $\lim_{x\to -3} \frac{4x^2+12x}{5x+15}$
P : *What is your method/strategy to solve the problem?*

A25	:	*First, find the value of* $\lim_{x\to 2}\frac{1+x-\sqrt{x^2+x+3}}{2-x}$ *by multiplying irrational root friend. Second, find the value of* $\lim_{x\to -3}\frac{4x^2+12x}{5x+15}$ *by factoring. Third, add the values of strategy 1 and strategy 2, namely* $\lim_{x\to 2}\frac{1+x-\sqrt{x^2+x+3}}{2-x}$ *plus* $\lim_{x\to -3}\frac{4x^2+12x}{5x+15}$
P	:	*For your strategy, is it written or not?*
A25	:	*Yes miss, it is written.*
P	:	*What is your process for solving the problem as a whole?*
A25	:	*First of all ... I tried to put the value of x into the limit equation. The result is* $\frac{0}{0}$*. So the form of is indeterminate ... if the result is* $\frac{0}{0}$ *... another method must be used ... use the method multiplied by the irrational root friend/factor .... so that the result isn't* $\frac{0}{0}$*. So for the first limit equation,,* $\lim_{x\to 2}\frac{1+x-\sqrt{x^2+x+3}}{2-x}$ *I multiply irrational root friend, then we get* $-\frac{1}{6}$*. It's first equation. Then, second equation ... find* $\lim_{x\to -3}\frac{4x^2+12x}{5x+15}$ *. I factor it, the result* $\lim_{x\to -3}\frac{4x(x+3)}{5(x+3)}$ *. is* $-\frac{12}{5}$*. Then, I add first equation and second equation, namely* $\lim_{x\to 2}\frac{1+x-\sqrt{x^2+x+3}}{2-x}+\lim_{x\to -3}\frac{4x^2+12x}{5x+15}$ *. The result is* $\frac{-5-72}{30}=-\frac{77}{30}$ *simplified to* $-2\frac{17}{30}$
P	:	*Did you carry out the steps of solving the problem according to the strategy you defined?*
A25	:	*Yes ... I did*
P	:	*Did you check your solution process and answer?*
A25	:	*I did*
P	:	*How did you check the process?*
A25	:	*Way to check, check the strategy again. There is something wrong with the calculation or not.*
P	:	*What is the answer you get?*
A25	:	*The result of the sum of the limits is* $-2\frac{17}{30}$

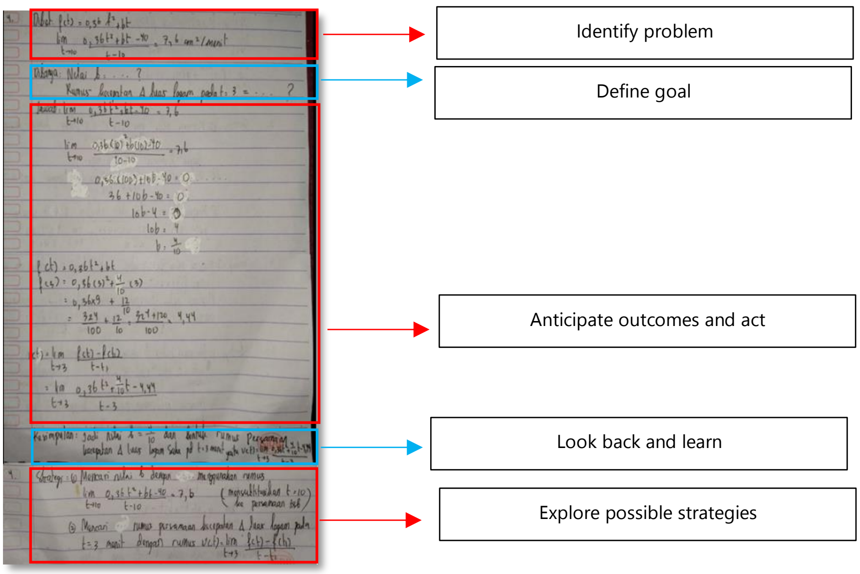

Figure 12. Results of Student A25's Test on Problem Number 4

Figure 12 shows the results of student A25's work on problem number 4, it can be seen that student A25 fulfill all IDEAL solution steps. The following is the transcript of student A25's work on the problem solving number 4.

P	:	*What information do you know about the problem?*
A25	:	*It's known that* $f(t)$ *is* $0{,}36t^2+bt$*. Then we know the formula* $v(t)=\lim_{t\to 10}\frac{0{,}36t^2+bt-40}{t-10}=7{,}6\ cm^2/minute$.
P	:	*What do you need to find in the problem?*
A25	:	*The value of b and the formula for the rate of change of the metal area when t = 3.*
P	:	*How do you solve the problem?*
A25	:	*The strategy is to evaluate b using the formula* $\lim_{t\to 10}\frac{0{,}36t^2+bt-40}{t-10}=7{,}6$ *. So, substitute t=10 into the limit equation. Then find the formula for the rate of change of the metal area at t =3 minutes using the formula* $v(t)=\lim_{t\to 3}\frac{f(t)-f(t_1)}{t-t_1}$.

P	:	*For your strategy, do you write it down or not?*
A25	:	*Yes miss.*
P	:	*What is your process for solving the problem as a whole?*
A25	:	*So the process is to find the value of b using the formula* $\lim_{t\to 10} \frac{0{,}36t^2+bt-40}{t-10} = 7{,}6$. *Then substitute t is close to 10 and the result is* $\frac{0{,}36(10)^2+b(10)-40}{10-10} = 7{,}6$. *Then* $36 + 10b - 40 = 0$. b *is generated* $\frac{4}{10}$. *Then, find* $f(t_1)$ *using formula* $f(t) = 0{,}36t^2 + bt$. t *replaced by 3....* $f(3) = 0{,}36(3)^2 + \frac{4}{10}(3) =$ *4,44. Then find the equation for the speed of change in metal area at t = 3 minutes using formula* $v(t) = \lim_{t\to 3} \frac{f(t)-f(t_1)}{t-t_1}$. $f(t)$ *is known to be* $0{,}36t^2 + \frac{4}{10}t$ *then* $f(t_1)$ *is known to be 4,44. Then divided by* $(t - 3)$. *So, the result is* $\lim_{t\to 3} \frac{0{,}36t^2+\frac{4}{10}t-4{,}44}{t-3}$
P	:	*Did you carry out the steps of solving the problem according to the strategy you defined?*
A25	:	*I did*
P	:	*Did you check your solution process and answer?*
A25	:	*No, but I was sure of the answer ... So, I didn't check it again.*
P	:	*What is the answer you got?*
A25	:	*Valuei b* $\frac{4}{10}$ *and the formula for the equation of the speed of change in Saka's metal area at t = 3 minutes is* $v(t) = \lim_{t\to 3} \frac{0{,}36t^2+\frac{4}{10}t-4{,}44}{t-3}$.

Based on the results of the analysis of test data and interviews with students A24 and A25, students with high learning independence in solving problems fulfill all indicators of IDEAL steps. Students fulfill identify problem indicator because students are able to write and mention information in the problem correctly. This is consistent with research of Abidin, Rodliyah, and Syaifuddin's (2021) that students with high learning independence can write and mention information in the problem correctly. In the define goal indicator, students are able to write and mention things that must be found in the problem. In the explore possible strategies indicator, students are able to plan a solution strategy for the problem. This is in line with the research of Ekadiarsi and Khusna (2022), where students with high independence are able to pass the stage of determining solution strategy to solve problem.

In the anticipate outcomes and act indicator, students are able to write down solution process for the solution steps taken until find answer of the problem. This is in line with Suciati (2016), students with high learning independence are able to solve problems by using the right steps. In the indicator of look back and learn, students tend to do process of checking the completion of the problem such as rechecking working steps, calculation operations, and making conclusions for problems. During problem solving process, students with high learning independence can problem solving well. This is in line with research conducted by Sundayana (2016) which states that the higher student's learning independence, so have higher problem solving ability.

Based on the presentation of the results from researcher's research about mathematical problem solving ability in terms of student learning independence with IDEAL Step in class XI MAN 1 Kota Kediri, it is found that students' problem solving ability is very diverse when viewed from learning independence. There are students who are able to solve problems well and there are also students who haven't been able to solve problems well. The benefits of using IDEAL steps in this study are it can be used to determine the level of ability and flow of student's problem solving from stages of identify problem, define goal, solving strategies, solving processes, and can see how students look back and learn at the problem solving process. Apart from that, with IDEAL steps can also be known forms of student errors in problem solving such as in terms of solving actions, writing, solving strategies, and implementing the flow of solving strategies to further stages.

CONCLUSION

The results showed that there are 3 categories of students' mathematical problem solving ability in terms of learning independence, namely low, medium and high. Students with low learning independence in solving problems only fulfill 2 indicators of IDEAL steps. Students with medium and high learning independence fulfill all indicators of IDEAL solution steps. Suggestions for future researchers is future researchers can conduct further research with different research variables or analyze problem solving errors in relation to limit functions with IDEAL steps.

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